Bank Crashes and Micro Enterprise *Loans*

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Abstract

This paper begins with an analysis of trends - over the period 2012-2018 - for total bank loans, non-performing loans and the number of active, working enterprises. A review survey was done on national data from Italy with a comparison developed on a local subset from the Sardinian Region. Empirical evidence appears to support the hypothesis of the paper: can the rating class assigned by banks - using current IRB and A-IRB systems - to micro and very small enterprises, whose ability to replace financial resources using endogenous means is structurally impaired, ipso facto orient the results of performance in the same terms of PD – Probability of Default assigned by the algorithm, thereby upending the principle of cause and effect? The thesis is developed through mathematical modelling that demonstrates the interaction of the measurement tool (the rating algorithm applied by banks) on the collapse of the loan status (default, performing or some intermediate point) of the assessed micro-entity. Emphasis is given, in conclusion, to the phenomenon using evidence of the intrinsically mutualistic link of the two populations of banks and (micro) enterprises provided by a system of differential equations.

Keywords: credit big data, rating models, MSE (Micro Small Enterprises) lending, financial system stability

JEL Codes: C02; C18; G24

Introduction: empirical evidence

Firstly, trend data (for the years 2012-2018) was collected and organised on the number of enterprises in the Sardinian regional and Italian national production sectors and, over the same interval, on their performance trends from the credit and financial system.Necessarily, a selection of the graphs, with which the analyses was observed, that provide reasoning for this workare included below.

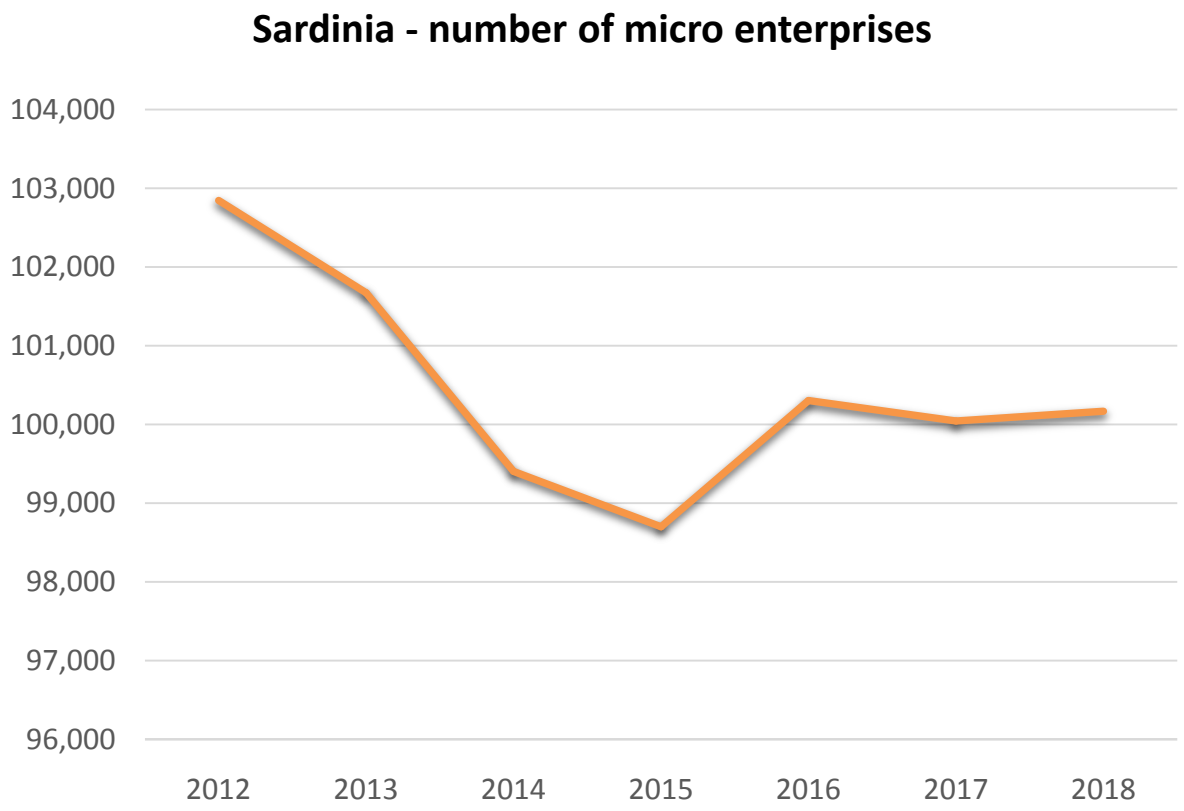
Section figures 1 and 2 - Number of micro enterprises (normalization from ISTAT and Chambers of Commerce sources[1]*)*

Figure 1

Dataset: number of micro enterprises in Sardinia							
Year	2012	2013	2014	2015	2016	2017	2018
B: extraction of minerals from quarries and mines	117	112	97	93	94	102	102
C: manufacturing	7,492	7,283	6,886	6,814	6,828	7,023	7,031
D: supply of electricity, gas, steam and air conditioning	85	109	107	113	132	109	109
E: supply of water, sewerage, waste management and environmental remediation services	190	198	199	214	207	201	201
F: construction	14,340	13,773	13,121	12,619	12,639	13,227	13,243
G: wholesale and retail trade, repair of motor vehicles and motorcycles	30,252	30,137	29,205	28,653	28,993	29,290	29,326
H: transport and storage	3,090	3,038	2,910	2,807	2,853	2,924	2,927
I: accommodation and food service businesses	9,409	9,521	9,424	9,499	9,795	9,478	9,490
J: information and communications services	1,856	1,835	1,783	1,801	1,821	1,809	1,812
K: financial and insurance service businesses	1,613	1,612	1,634	1,637	1,668	1,624	1,626
L: real estate businesses	2,776	2,946	2,860	2,863	3,016	2,877	2,880
M: professional, scientific and technical businesses	15,880	15,445	15,375	15,552	15,880	15,542	15,562
N: rental and travel agencies, business support services	3,313	3,165	3,135	3,141	3,187	3,171	3,175
P: education	528	537	532	533	535	530	531
Q: healthcare and social services	6,161	6,218	6,422	6,612	6,713	6,391	6,399
R: arts, sports, entertainment and amusement businesses	1,217	1,193	1,178	1,146	1,207	1,182	1,183
S: other service businesses	4,528	4,552	4,536	4,604	4,737	4,567	4,572
TOTAL	**102,847**	**101,674**	**99,404**	**98,701**	**100,305**	**100,045**	**100,169**

Figure 2

years/number of micro enterprises in Sardinia vectors	2012	2013	2014	2015	2016	2017	2018
	102,847	101,674	99,404	98,701	100,305	100,045	100,169

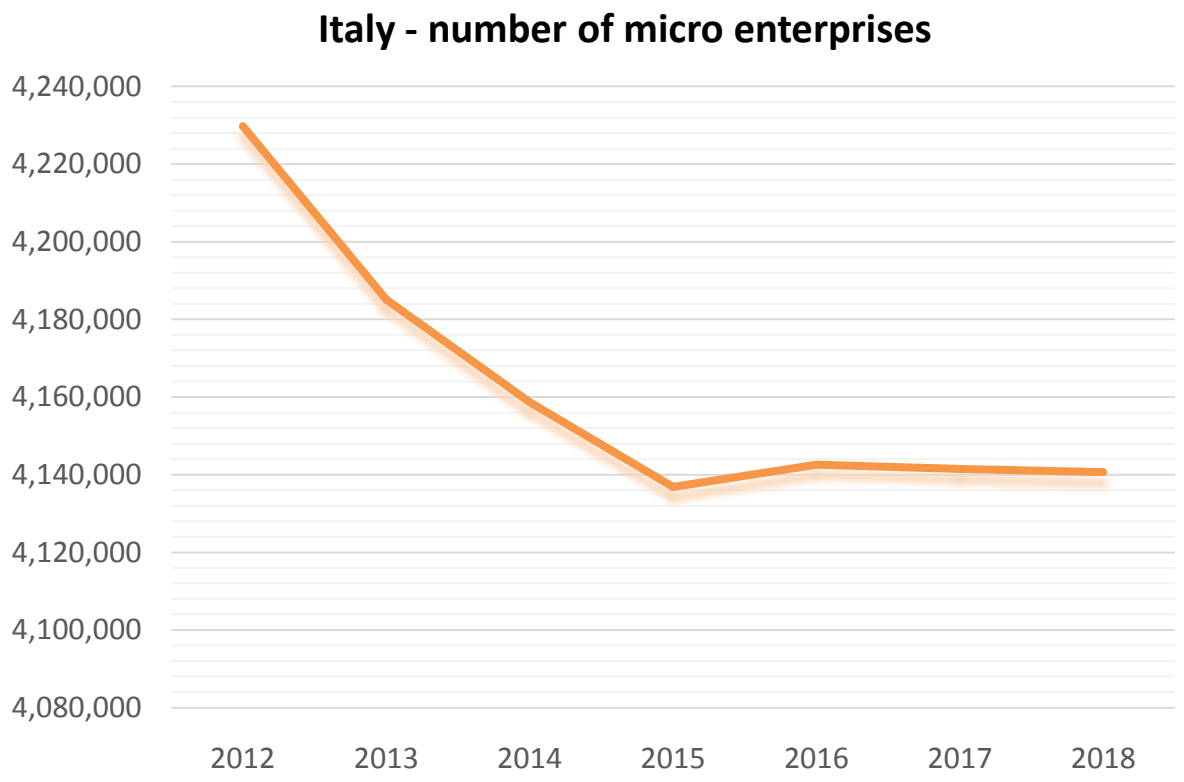


Dataset: number of micro enterprises in Italy							
Year	2012	2013	2014	2015	2016	2017	2018
B: extraction of minerals from quarries and mines	1,907	1,850	1,775	1,712	1,796	1,795	1,795
C: manufacturing	345,293	338,015	328,486	321,837	330,613	330,526	330,459
D: supply of electricity, gas, steam and air conditioning	8,380	9,610	9,916	10,205	9,448	9,445	9,443
E: supply of water, sewerage, waste management and environmental remediation services	6,485	6,688	6,748	6,816	6,628	6,626	6,625
F: construction	548,709	528,592	509,648	492,388	515,477	515,341	515,237
G: wholesale and retail trade, repair of motor vehicles and motorcycles	1,124,546	1,116,087	1,086,631	1,068,659	1,089,768	1,089,481	1,089,262
H: transport and storage	119,126	117,430	113,241	110,756	114,173	114,143	114,120
I: accommodation and food service businesses	288,119	294,007	292,996	295,706	290,253	290,177	290,119
J: information and communications services	91,274	89,895	91,020	92,279	90,353	90,329	90,311

K: financial and insurance service businesses	88,998	90,637	92,831	93,799	90,799	90,775	90,757
L: real estate businesses	234,738	242,874	238,492	237,637	236,437	236,374	236,327
M: professional, scientific and technical businesses	702,053	683,778	698,154	707,020	691,902	691,720	691,581
N: rental and travel agencies, business support services	132,452	128,082	128,721	128,394	128,327	128,294	128,268
P: education	25,239	25,957	27,351	27,781	26,359	26,352	26,347
Q: healthcare and social services	253,160	254,655	270,894	278,646	262,123	262,054	262,001
R: arts, sports, entertainment and amusement businesses	60,658	60,382	62,001	63,011	60,997	60,981	60,969
S: other service businesses	198,593	196,542	199,755	200,185	197,103	197,051	197,011
TOTAL	**4,229,730**	**4,185,081**	**4,158,660**	**4,136,831**	**4,142,556**	**4,141,465**	**4,140,633**

Figure 4

years/number of micro enterprises in Italy vectors	2012	2013	2014	2015	2016	2017	2018
	4,229,730	4,185,081	4,158,660	4,136,831	4,142,556	4,141,465	4,140,633



Section figures 3 and 4 - Number of macro enterprises (normalization from ISTAT and Chambers of Commerce sources)

Figure 5

Dataset: number of macro enterprises in Sardinia							
Year	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**
B: extraction of minerals from quarries and mines	35	32	30	30	28	33	34
C: manufacturing	658	607	566	521	537	619	625
D: supply of electricity, gas, steam and air conditioning	12	14	12	11	11	13	13
E: supply of water, sewerage, waste management and environmental remediation services	96	95	84	87	80	95	96
F: construction	514	448	375	365	371	444	448
G: wholesale and retail trade, repair of motor vehicles and motorcycles	792	774	712	712	729	796	805
H: transport and storage	266	259	251	254	274	279	282
I: accommodation and food service businesses	469	437	438	460	527	499	504
J: information and communications services	67	69	76	73	68	76	76
K: financial and insurance service businesses	29	31	27	25	31	31	31
L: real estate businesses	19	12	7	9	7	12	12
M: professional, scientific and technical businesses	79	78	74	74	84	83	84
N: rental and travel agencies, business support services	259	244	250	233	251	265	268
P: education	31	29	33	34	38	35	36
Q: healthcare and social services	284	294	302	301	318	321	324
R: arts, sports, entertainment and amusement businesses	69	62	62	62	71	70	71
S: other service businesses	72	72	71	65	72	75	76
TOTAL	**3,751**	**3,557**	**3,370**	**3,316**	**3,497**	**3,746**	**3,784**

Figure 6

years/number of Macro enterprises in Sardinia vectors	2012	2013	2014	2015	2016	2017	2018
	3,751	3,557	3,370	3,316	3,497	3,746	3,784

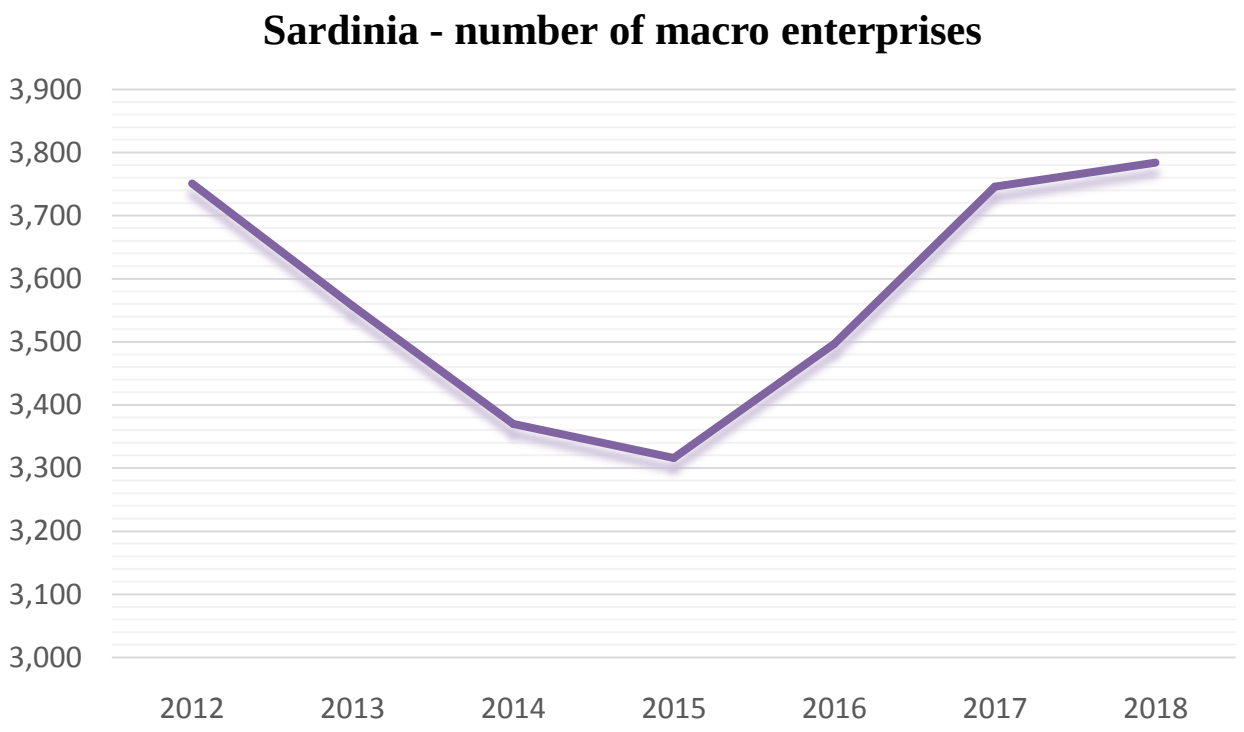
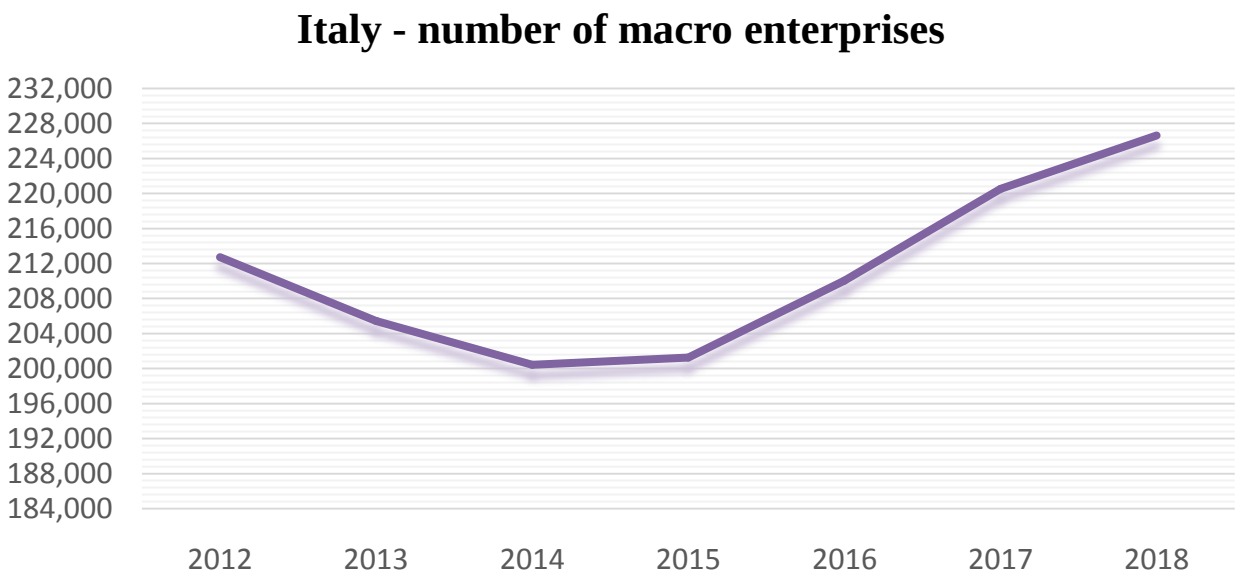


Figure 7

Dataset: number of macro enterprises in Italy							
Year	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**
B: extraction of minerals from quarries and mines	544	486	482	474	454	522	537
C: manufacturing	72,013	69,329	67,936	67,480	68,845	74,003	76,049
D: supply of electricity, gas, steam and air conditioning	546	559	543	570	567	596	613
E: supply of water, sewerage, waste management and environmental remediation services	2,482	2,433	2,398	2,415	2,432	2,604	2,676
F: construction	23,703	21,254	19,455	19,017	19,347	22,007	22,616
G: wholesale and retail trade, repair of motor vehicles and motorcycles	38,867	37,553	36,503	36,568	38,349	40,221	41,334
H: transport and storage	12,629	12,435	12,447	12,869	13,478	13,674	14,052
I: accommodation and food service businesses	19,759	19,200	19,017	19,758	21,747	21,301	21,890
J: information and communications services	6,006	6,094	5,977	6,102	6,580	6,586	6,768
K: financial and insurance service businesses	2,436	2,394	2,378	2,374	2,400	2,566	2,637
L: real estate businesses	696	690	642	636	700	720	740
M: professional, scientific and technical businesses	7,964	7,922	7,741	7,914	8,566	8,588	8,825
N: rental and travel agencies, business support services	11,318	11,280	11,177	11,201	11,632	12,121	12,456
P: education	1,651	1,720	1,737	1,785	2,001	1,904	1,957
Q: healthcare and social services	6,240	6,401	6,401	6,585	7,047	6,996	7,190
R: arts, sports, entertainment and amusement businesses	2,396	2,322	2,168	2,011	2,168	2,369	2,435
S: other service businesses	3,472	3,360	3,425	3,495	3,728	3,743	3,846
TOTAL	**212,722**	**205,432**	**200,427**	**201,254**	**210,041**	**220,523**	**226,621**

Figure 8

years/number of macro enterprises in Italy vectors	2012	2013	2014	2015	2016	2017	2018
	212,722	205,432	200,427	201,254	210,041	220,523	226,621



***** *Section figures 5 and 6 - Loans to micro enterprises (self-processed data of Bank of Italy - stock values in thousands of euro)*

Figure 9

Sardinia - Monthly surveys of loans to micro enterprises - stock values in thousands of euro							
31/01/2012	3,924,505	31/10/2013	3,596,882	31/07/2015	3,523,998	30/04/2017	3,443,130
29/02/2012	3,919,951	30/11/2013	3,566,020	31/08/2015	3,505,632	31/05/2017	3,452,826
31/03/2012	3,865,810	31/12/2013	3,556,005	30/09/2015	3,516,844	30/06/2017	3,399,004
30/04/2012	3,876,134	31/01/2014	3,644,238	31/10/2015	3,498,269	31/07/2017	3,305,219
31/05/2012	3,880,775	28/02/2014	3,633,203	30/11/2015	3,536,971	31/08/2017	3,294,734
30/06/2012	3,853,389	31/03/2014	3,628,280	31/12/2015	3,509,561	30/09/2017	3,291,878
31/07/2012	3,819,758	30/04/2014	3,613,463	31/01/2016	3,505,739	31/10/2017	3,305,199
31/08/2012	3,795,494	31/05/2014	3,579,649	29/02/2016	3,518,556	30/11/2017	3,325,509
30/09/2012	3,778,401	30/06/2014	3,570,033	31/03/2016	3,521,410	31/12/2017	3,305,659
31/10/2012	3,786,101	31/07/2014	3,548,786	30/04/2016	3,507,487	31/01/2018	3,311,735
30/11/2012	3,779,619	31/08/2014	3,518,787	31/05/2016	3,518,370	28/02/2018	3,296,940
31/12/2012	3,722,310	30/09/2014	3,534,184	30/06/2016	3,493,093	31/03/2018	3,304,006
31/01/2013	3,731,535	31/10/2014	3,527,510	31/07/2016	3,438,590	30/04/2018	3,307,596
28/02/2013	3,722,496	30/11/2014	3,522,209	31/08/2016	3,435,170	31/05/2018	3,318,292
31/03/2013	3,705,210	31/12/2014	3,519,180	30/09/2016	3,421,025	30/06/2018	3,029,209

30/04/2013	3,709,212	31/01/2015	3,527,181	31/10/2016	3,413,568	31/07/2018	3,016,395
31/05/2013	3,704,006	28/02/2015	3,518,679	30/11/2016	3,429,177	31/08/2018	3,004,079
30/06/2013	3,662,903	31/03/2015	3,545,684	31/12/2016	3,410,455	30/09/2018	2,995,745
31/07/2013	3,634,410	30/04/2015	3,544,313	31/01/2017	3,431,125	31/10/2018	3,018,993
31/08/2013	3,601,313	31/05/2015	3,526,313	28/02/2017	3,442,245	30/11/2018	3,041,101
30/09/2013	3,593,634	30/06/2015	3,557,350	31/03/2017	3,441,119	31/12/2018	2,895,286

Figure 10

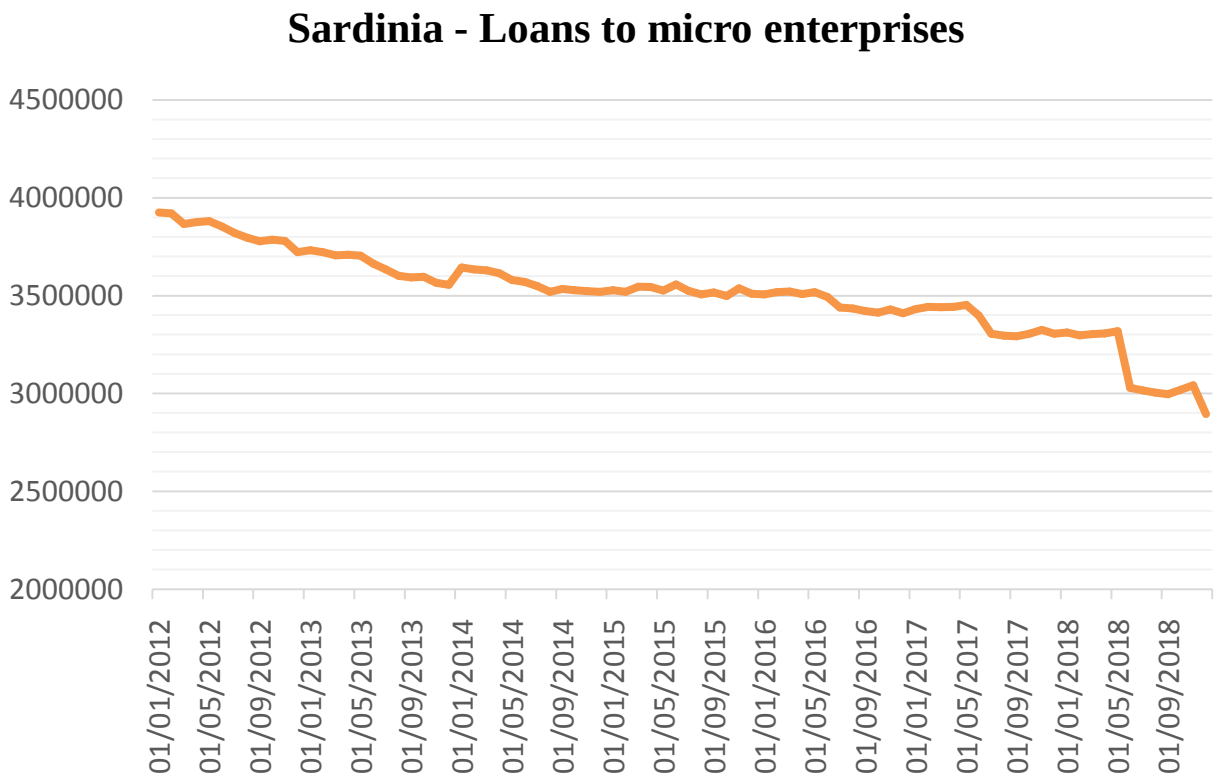
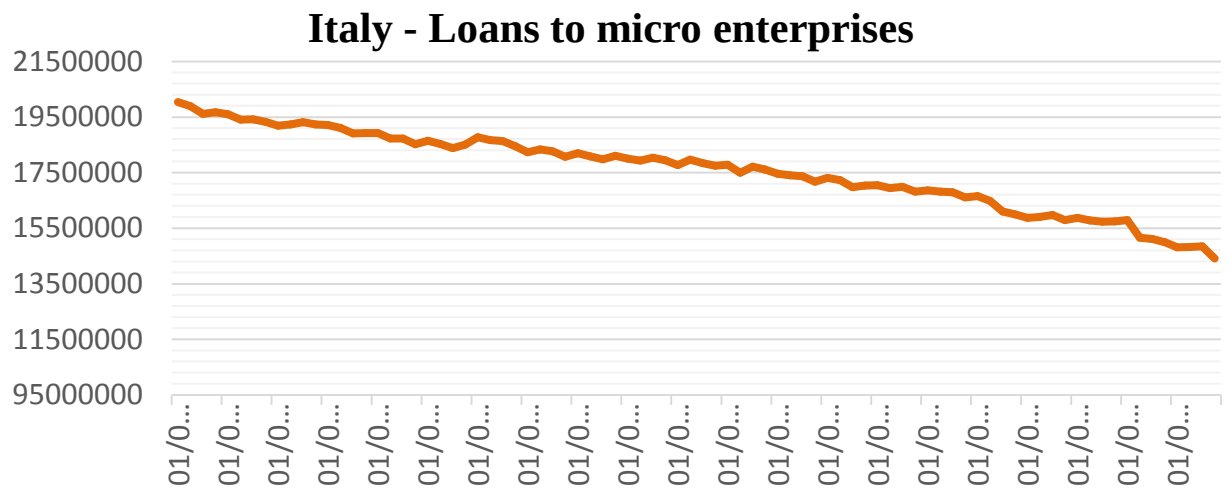


Figure 11

Italy: quarterly surveys of loans to micro enterprises - stock values in thousands of euro							
31/03/2012	196,104,283	31/12/2013	185,086,851	30/09/2015	177,848,224	30/06/2017	164,846,165
30/06/2012	194,108,548	31/03/2014	186,303,319	31/12/2015	176,092,382	30/09/2017	158,686,183
30/09/2012	191,839,155	30/06/2014	183,362,737	31/03/2016	173,619,544	31/12/2017	157,869,518
31/12/2012	192,277,603	30/09/2014	181,991,189	30/06/2016	172,330,425	31/03/2018	157,294,801
31/03/2013	189,075,414	31/12/2014	180,982,518	30/09/2016	170,408,469	30/06/2018	151,569,685
30/06/2013	187,226,274	31/03/2015	180,320,433	31/12/2016	168,192,831	30/09/2018	148,124,634
30/09/2013	186,463,427	30/06/2015	179,629,410	31/03/2017	167,930,164	31/12/2018	144,068,531

Figure 12



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Section figures 7 and 8 - Loans to macro enterprises (self-processed data of Bank of Italy- stock values in thousands of euro)

Figure 13

Sardinia: monthly surveys of loans to macro enterprises - stock values in thousands of euro							
31/01/2012	20,945,376	31/10/2013	18,626,870	31/07/2015	19,223,444	30/04/2017	18,641,376
29/02/2012	20,782,407	30/11/2013	18,294,656	31/08/2015	19,175,133	31/05/2017	18,783,067
31/03/2012	19,755,188	31/12/2013	18,043,543	30/09/2015	19,032,800	30/06/2017	17,918,440
30/04/2012	20,148,404	31/01/2014	19,254,395	31/10/2015	18,949,734	31/07/2017	17,910,313
31/05/2012	19,926,114	28/02/2014	19,358,613	30/11/2015	19,003,366	31/08/2017	17,738,670
30/06/2012	19,240,685	31/03/2014	18,982,302	31/12/2015	19,099,174	30/09/2017	17,243,961
31/07/2012	19,482,460	30/04/2014	19,752,931	31/01/2016	19,112,655	31/10/2017	17,560,839
31/08/2012	19,184,105	31/05/2014	19,601,713	29/02/2016	19,015,982	30/11/2017	17,261,327
30/09/2012	19,126,712	30/06/2014	18,825,923	31/03/2016	19,080,784	31/12/2017	16,993,650
31/10/2012	19,253,365	31/07/2014	18,969,477	30/04/2016	19,130,911	31/01/2018	16,970,089
30/11/2012	19,423,965	31/08/2014	19,579,825	31/05/2016	19,202,102	28/02/2018	17,048,694
31/12/2012	18,902,468	30/09/2014	18,596,942	30/06/2016	19,226,436	31/03/2018	16,844,715
31/01/2013	19,268,629	31/10/2014	19,050,007	31/07/2016	18,919,625	30/04/2018	17,179,620
28/02/2013	19,238,602	30/11/2014	18,619,485	31/08/2016	18,724,079	31/05/2018	17,632,867
31/03/2013	18,896,487	31/12/2014	18,416,271	30/09/2016	18,765,656	30/06/2018	16,543,379
30/04/2013	19,209,658	31/01/2015	18,800,787	31/10/2016	18,805,788	31/07/2018	15,872,481
31/05/2013	18,891,729	28/02/2015	18,647,498	30/11/2016	18,629,860	31/08/2018	15,763,331

30/06/2013	18,818,546	31/03/2015	18,962,928	31/12/2016	18,339,360	30/09/2018	14,822,036
31/07/2013	18,726,947	30/04/2015	18,963,265	31/01/2017	18,799,061	31/10/2018	15,383,655
31/08/2013	18,832,875	31/05/2015	18,900,447	28/02/2017	18,727,914	30/11/2018	15,390,497
30/09/2013	18,288,769	30/06/2015	19,195,611	31/03/2017	17,952,327	31/12/2018	14,189,483

Figure 14

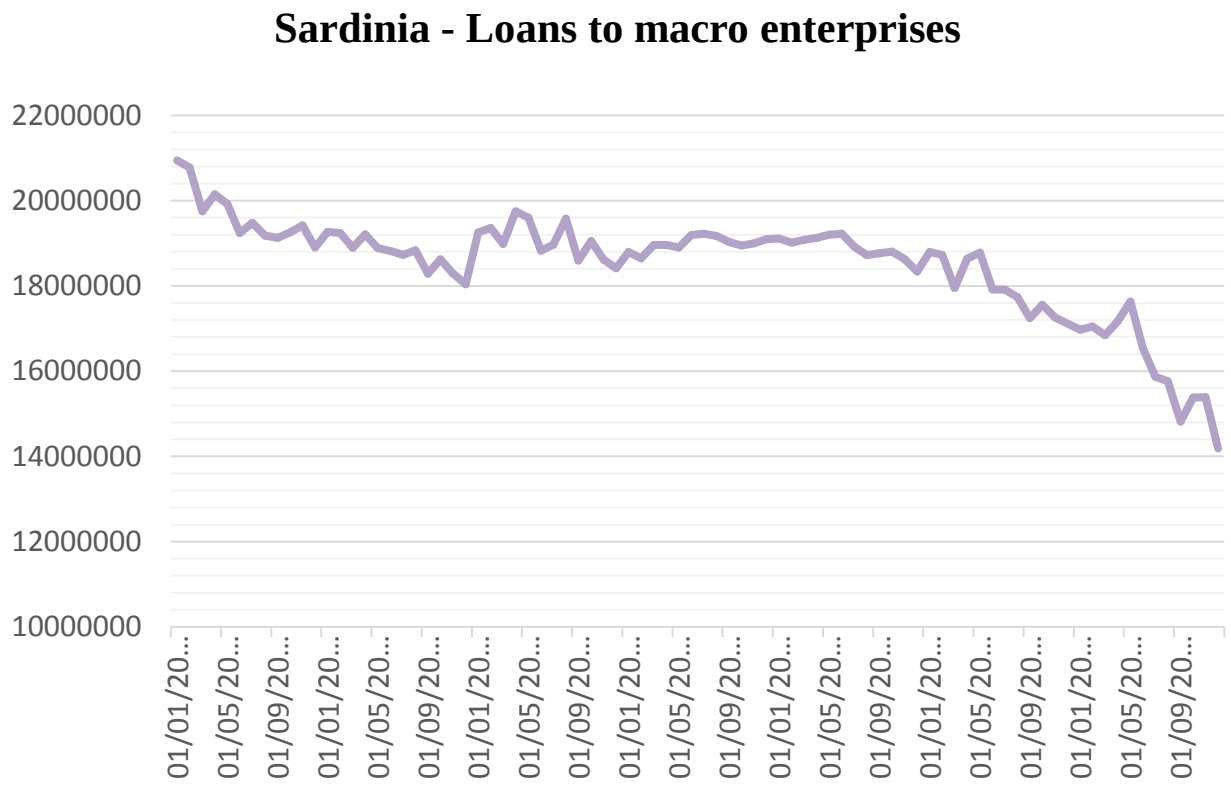
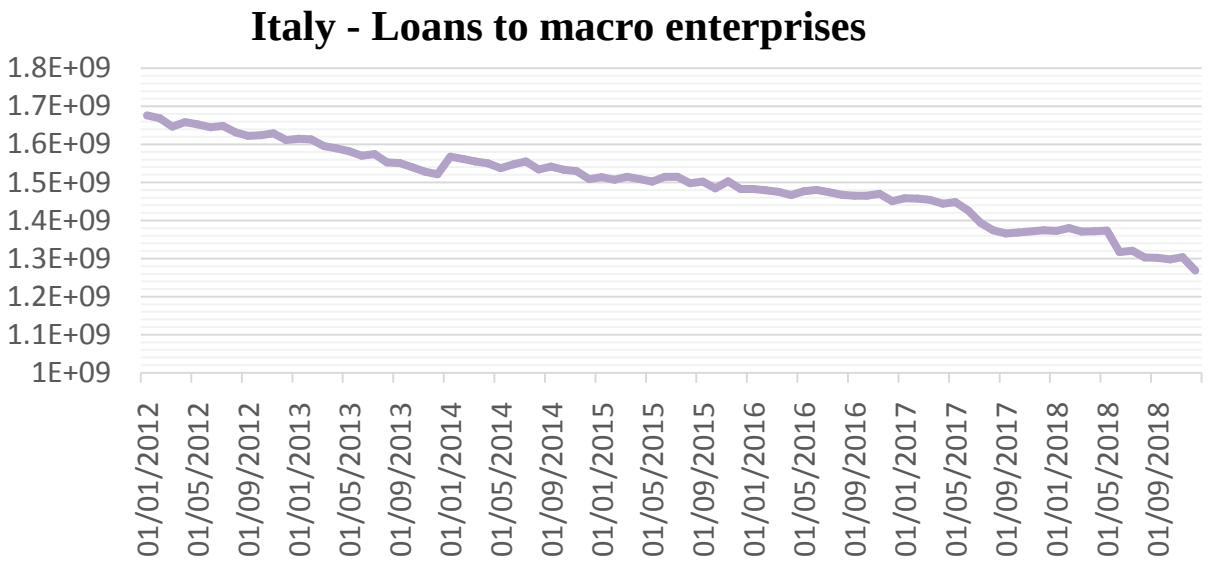


Figure 15

Italy: quarterly surveys of loans to macro enterprises - stock values in thousands of euro							
31/03/2012	1,646,994,366	31/12/2013	1,521,713,568	30/09/2015	1,501,913,382	30/06/2017	1,426,588,448
30/06/2012	1,645,185,055	31/03/2014	1,553,078,446	31/12/2015	1,483,096,671	30/09/2017	1,366,124,098
30/09/2012	1,622,500,182	30/06/2014	1,547,273,266	31/03/2016	1,475,216,267	31/12/2017	1,374,039,081
31/12/2012	1,611,501,427	30/09/2014	1,541,493,794	30/06/2016	1,480,433,998	31/03/2018	1,370,734,809
31/03/2013	1,595,250,768	31/12/2014	1,509,099,932	30/09/2016	1,465,279,100	30/06/2018	1,316,758,282
30/06/2013	1,570,164,850	31/03/2015	1,514,608,082	31/12/2016	1,450,690,989	30/09/2018	1,301,863,430
30/09/2013	1,550,624,325	30/06/2015	1,514,910,903	31/03/2017	1,454,317,652	31/12/2018	1,268,272,037

Figure 16



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Section figures 9 and 10 - Net non-performing loans to micro enterprises (self-processed data of Bank of Italy - stock values in thousands of euro)

Figure 17

Sardinia: net non performing loans to micro enterprises - in thousands of euro					
31/03/2012	415	**31/12/2013**	424	**30/09/2015**	488
30/06/2012	421	**31/03/2014**	434	**31/12/2015**	548
30/09/2012	429	**30/06/2014**	436	**31/03/2016**	557
31/12/2012	442	**30/09/2014**	443	**30/06/2016**	551
31/03/2013	405	**31/12/2014**	453	**30/09/2016**	523
30/06/2013	412	**31/03/2015**	464	**31/12/2016**	524
30/09/2013	408	**30/06/2015**	479	**31/03/2017**	514

Figure 18

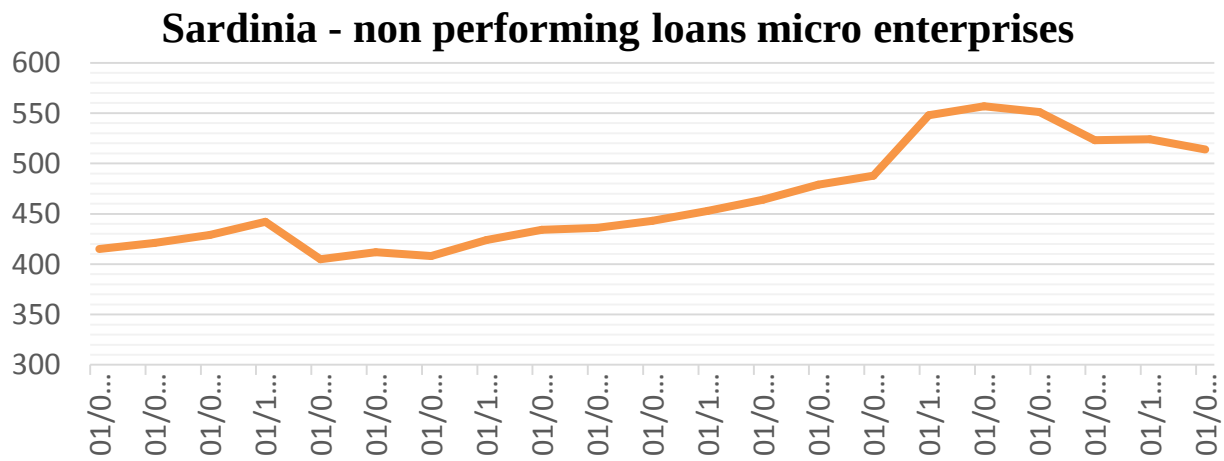
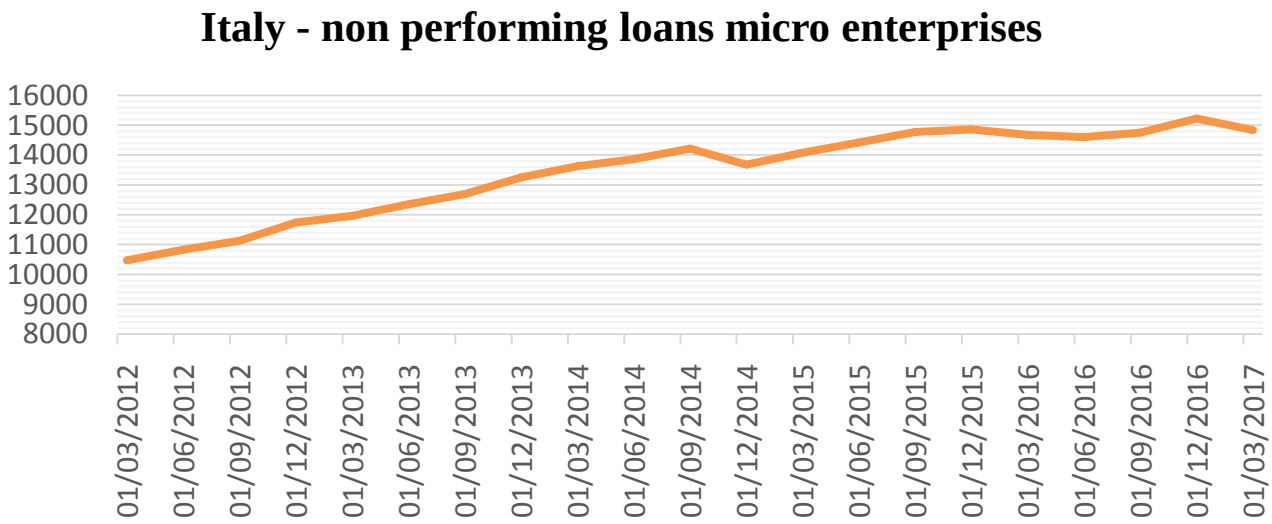


Figure 19

Italy: net non performing loans to micro enterprises - in thousands of euro					
31/03/2012	10,473	31/12/2013	13,253	30/09/2015	14,774
30/06/2012	10,825	31/03/2014	13,624	31/12/2015	14,859
30/09/2012	11,126	30/06/2014	13,869	31/03/2016	14,683
31/12/2012	11,743	30/09/2014	14,214	30/06/2016	14,603
31/03/2013	11,967	31/12/2014	13,680	30/09/2016	14,754
30/06/2013	12,350	31/03/2015	14,076	31/12/2016	15,229
30/09/2013	12,692	30/06/2015	14,425	31/03/2017	14,845

Figure 20



*****Section figures 11 and 12 - Net non-performing loans to macro enterprises (*self-processed data of Bank of Italy* - stock values in thousands of euro)

'Figure 21

Sardinia: net non performing loans to macro enterprises - in thousands of euro					
31/03/2012	1,455	31/12/2013	1,609	30/09/2015	2,209
30/06/2012	1,549	31/03/2014	1,681	31/12/2015	2,814
30/09/2012	1,627	30/06/2014	1,774	31/03/2016	2,834
31/12/2012	1,618	30/09/2014	1,908	30/06/2016	2,832
31/03/2013	1,334	31/12/2014	2,057	30/09/2016	2,815
30/06/2013	1,405	31/03/2015	2,045	31/12/2016	2,952
30/09/2013	1,499	30/06/2015	2,137	31/03/2017	2,938

Figure 22

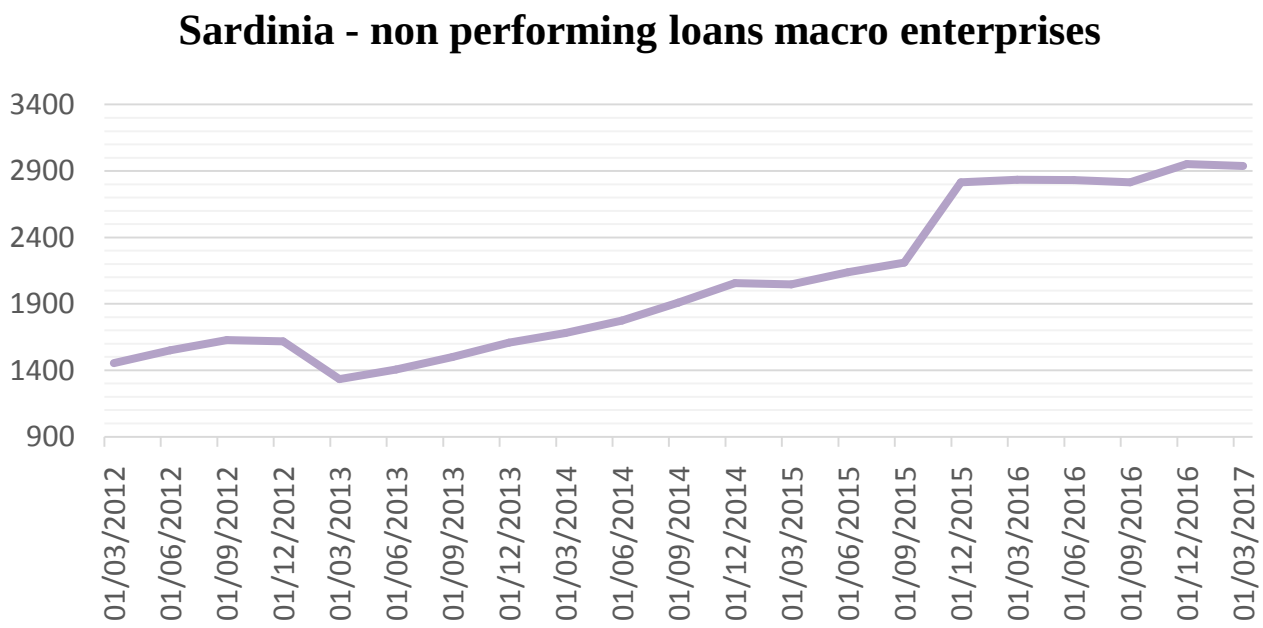
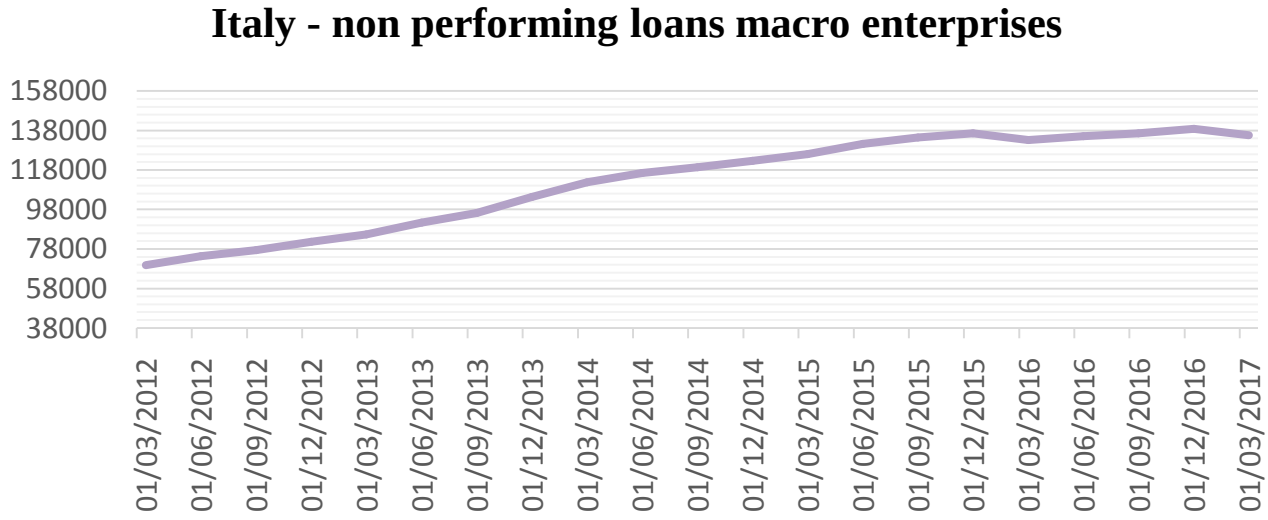


Figure 23

Italy: net non performing loans macro enterprises - in thousands of euro					
31/03/2012	69,899	**31/12/2013**	104,258	**30/09/2015**	134,512
30/06/2012	74,349	**31/03/2014**	111,726	**31/12/2015**	136,564
30/09/2012	77,502	**30/06/2014**	116,406	**31/03/2016**	133,185
31/12/2012	81,677	**30/09/2014**	119,310	**30/06/2016**	135,080
31/03/2013	85,363	**31/12/2014**	122,643	**30/09/2016**	136,487
30/06/2013	91,292	**31/03/2015**	126,021	**31/12/2016**	138,805
30/09/2013	96,203	**30/06/2015**	131,237	**31/03/2017**	135,640

Figure 24



Description of the data

The scenario presented above, begins from the progressive reduction in the disbursement of loans - which we assume to be an independent variable - to which a generalised increase in net non-performing loans can be reasonably correlated[2] (average Bravais-Pearson correlation coefficient C_{BP}=-*0.57*). Nevertheless, the repercussion of this manifestation had different results on the size of the productive sector. On the one hand, micro enterprises tend to leave the market more quickly than *new*-companies of a similar size. At the same time, macro enterprises seem to not be affected by this phenomenon. Based on this observation, a theoretical model that explains the basis of these findings was developed.
From the data-set adopted, the correlation index between the reduction in the volumes of credit disbursed to the production system and the number of enterprises operating on the market in Italy was:+0.73 for micro enterprises and -0.71 for macro enterprises[3].

It should be taken into account that more than 90% of the makeup of Italian and Sardinian production comprises micro enterprises. In the overall statistical data, the granularity of even the largest enterprises is insufficient to compensate and support the general system, as the figures below show clearly.
Section figures 13 and 14 - Total number of enterprises (normalization from ISTAT and Chambers of Commerce sources)

Figure 25

	years/number of enterprises in Sardinia vectors						
	2012	2013	2014	2015	2016	2017	2018
Micro enterprises	102,847	101,647	99,404	98,701	100,305	100,045	100,169
Macro enterprises	3,751	3,557	3,370	3,316	3,497	3,746	3,784
Total enterprises	106,598	105,204	102,774	102,017	103,802	103,791	103,953

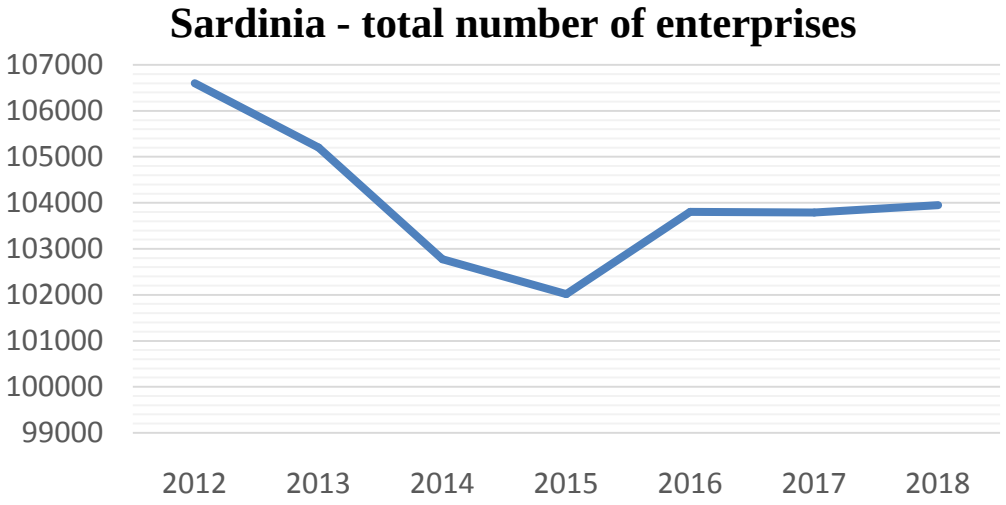
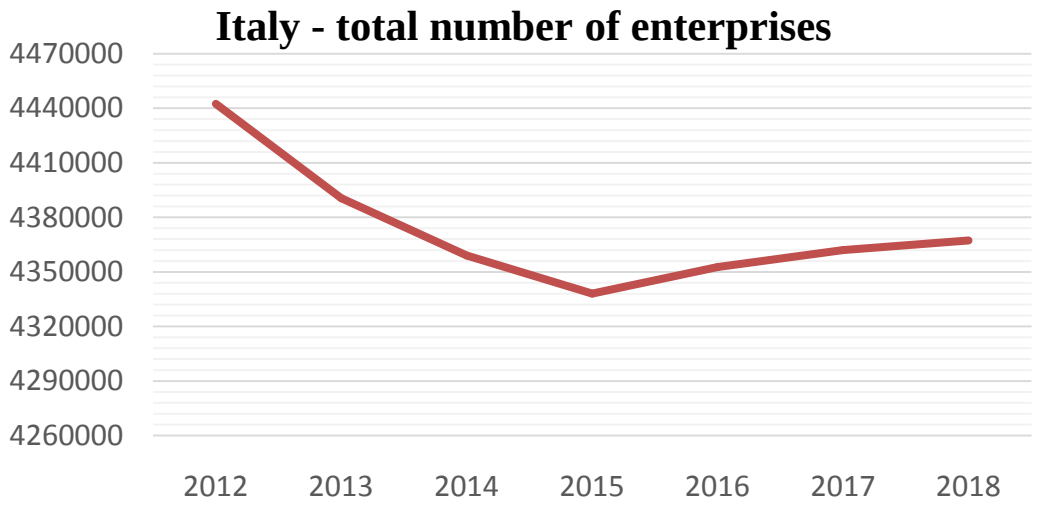


Figure 26

	years/number of enterprises in Italy vectors						
	2012	2013	2014	2015	2016	2017	2018
Micro enterprises	212,722	205,432	200,427	201,254	210,041	220,523	226,621
Macro enterprises	4,229,730	4,185,081	4,158,660	4,136,831	4,142,556	4,141,465	4,140,633
Total enterprises	4,442,452	4,390,513	4,359,087	4,338,085	4,352,597	4,361,988	4,367,254



Therefore, for all the reasons detailed above, stability, or rather growth, in a real economy founded on micro and very small enterprises, is a function - though not univariate - of the average credit disbursed. Less available credit significant impacts the real economy, especially, in countries where the financial system is centred on banks.Bond and equity markets are underdeveloped and therefore cannot provide alternative resources, especially for small businesses[4]. The worsening of the conditions of access to credit, for indebted and vulnerable subjects, as could be the micro enterprises, can leads significant effects: as the reduction in consumption and investments in the real economy.The Internal Rating-based (IRB) "basic" or "advanced", applied by banks, to their customers, it is made without distinctions due to the size of the operator requesting financial support. The assigned rating class determines whether a costumer can access credit risk and how much it will cost. The assigned rating class determines whether anenterprise can access credit and how much it will cost. This therefore orients the performance results for economic micro-entities, structurally unable to replace financial resources with endogenous means, in the same way as the Probability of Default (PD) assigned by the algorithm, thus reversing the cause and effect principle. In the next section we formalize this statement from a mathematical point of view.

The model

Banks implement their strategies for making loans based on commercial and profit-based rationales. Loans are represented by risk portfolios whose overall weighting is derived from the sum of segmented portions, which in turn are based on precise evaluations of each individual counterparty. In the model below, we will proceed by approximating the portfolio average and imagining that all the *n* positions comprising it[5] have the same PD, the same Loss Given Default(*LGD*)and the same, Exposure at Default (*EAD*), the total expected losses:TEL_n will be[6]:

$$TEL_n = n * PD * EAD * LGD (1)$$

Because these are expected losses, they represent a cost and should be recorded in the bank's Income Statement. A separate rationale should be applied to unexpected losses, which represent a capital constraint for banks, and which determine the caution that they more frequently adopt when making loans.

At this point, we have chosen here to apply the "Vasicek model"[7], about which we are offering an integrative emanation, which describes the different mathematical effects of credit intermediation, with the subsequent disaggregation of the peculiarities of the micro and very small enterprises from the total[8]. Our portfolio comprises the *n* positions (financed enterprises) described above and the observation we conduct is over a time *t* = 1 year:

$$TL_n = \sum_{i=1}^{n} U_i LGD_i EAD_i (2)$$

Where:

TL_n is the total loss on the portfolio over the time *t*;LGD_i and EAD_i are the LGD and the EAD of the i-th company respectively;U_i is the Boolean indicator, which takes on a value of 1 if the i-th enterprise has reached default within time *t*, or the value 0 if the i-th enterprise has continued to be performing in the range considered.

Now, let us suppose that the default is essentially determined, for all the companies in the portfolio, by the size of the budget surplus ($S_i$for the i-th enterprise): below a certain size, the business does not hold up and goes into default. S_i represents a random variable, making it necessary to proceed with the description of its evolutionary process. Note that this is a Markovian process[9]: at the instant t_0 the future random value of S_i in t_1 depends only on the state of $S_i$in t_0 and not on what occurred previously.

Now, let us introduce the amount X – to which we assign great significance for the progress of our thesis – which represents a variable of macroeconomic context and which indicates the further scenario in which the enterprises in the portfolio operate. From this there will be derived an additional random element *Y*, whose variation ΔY in an interval Δt is the result of the sum of the variations of the elementary states of *Y* (independent among themselves) and whose variance is analogously the sum of the elementary variances: if $\Delta Y(\delta t)$ has a variance σ^2, over the entire interval ΔT, we will see a total variance of ΔY equal to $\sigma^2 \Delta t$ and, with the same probability distribution of the elementary variations and a corresponding standard deviation of $\sigma\sqrt{\Delta t}$. Therefore:

$$dY = X\sigma\sqrt{\Delta t} \quad (3)$$

With *X* being a random Gaussian variable.
The overall process is described by the stochastic differential equation:

$$dS_i = S_i(t) r_i dt + S_i(t) X_i \sigma_i \sqrt{dt}^{10} \quad (4)$$

Supplementing with Ito's lemma[11], we find:

$$S_i(T) = S(0) e^{r_i T - \frac{1}{2}\sigma_i^2 T + \sigma_i \sqrt{T} X_i} (5)$$

It emerges that the randomness of the process $S_i(T)$ is determined solely by X_i.
By analysing X_i, we can break it down into:

$$X_i = \alpha K + \beta e_i (6)$$

where *K* is the random variable.It comes from the macroeconomic context, while the e_i element indicates the random idiosyncratic factor of the individual counterparty. It should be mentioned that a certain component of e_i is however dependent on *K*.

Assuming that both of these are normal Gaussian variables (with the average = 0 and the variance = 1) that are independent among themselves, then:

$$\alpha^2 + \beta^2 = 1 \quad (7)$$

and

$$X_i = \alpha K + \sqrt{1 - \alpha^2} e_i (8)$$

With α known a priori, the above formula describes the process that leads to the default: that is, for X_i being less than a minimum M_i

$$PD_i = P\,r(X_i < M_i) (9)$$

hence,

$$PD_i = F(M_i) (10)$$

and

$$M_i = F^{-1}(PD_i) (11)$$

with *F* being the cumulative probability function of the normal Gaussian distribution.

The Deterministic Approach

Then, based on the above, a default state will be verified when:

$$\alpha K + \sqrt{1 - \alpha^2} e_i < F^{-1}(PD_i) \quad (12)$$

The e_i factor is peculiar and intrinsic to each enterprise, we can useitfor isolate micro enterprises with respect to medium-large enterprises. In fact, this factor can be more easily defined on the basis of deterministic considerations, which can be made on the general and accounting structures of those same medium-large enterprises, since they are obviously less dependent on exogenous components of their resources and therefore less sensitive to any turbulence in the economic situation and system.
Instead, for micro and very small enterprises, we assumethat the state of *e*is described by a wave function $\psi(r, t)$, which indicates the amplitude of probability to find the state, at time *t*, of the enterprise *i* in *r*. Where "*r*" is a generic point of the [κ, λ] interval, $\kappa = 0$ is the performance state and $\lambda = 1$, which represents default. Even though the *r* state is also "deterministic", in this case, the result of the measurement becomes probabilistic.

$$P(r, t) \propto |\psi(r, t)|^2 (13)$$

Explaining further, for simplicity's sake, let us consider a generic state $|\psi\rangle$[12] expressed by the linear combination of the two limit states $|\kappa\rangle$ and $|\lambda\rangle$ (Eigenstates):

$$|\psi\rangle = \frac{1}{\sqrt{2}}\{|\kappa\rangle + |\lambda\rangle\} \quad (14)$$

The measuring instrument is the algorithm used by a generic bank *B*in order to calculate the rating, where *i*representsthe micro-enterprise requesting a loan. The idiosyncratic component $e_i$of the micro and very small enterprises, can be evaluated at any time, in all points of the interval [κ,λ]. The measurement of a "macroscopic" rating may result in three positions:

– (0) indicated by the $|B_0\rangle$ state, if the instrument reads that the micro-enterprise's state is $|\kappa\rangle$;

– (1) indicated by the $|B_1\rangle$ state, if the instrument reads that the micro-enterprise's state is $|\lambda\rangle$;

– (0.5) characterizes the adjustment of the instrument before it is measured.

The interaction between the phenomenon and the measuring produces:

$$|\psi(0)\rangle_1 = |\kappa\rangle|B_{0,5}\rangle \Rightarrow |\psi(t)\rangle_1 = |\kappa\rangle|B_0\rangle \quad (15)$$

$$|\psi(0)\rangle_2 = |\lambda\rangle|B_{0,5}\rangle \Rightarrow |\psi(t)\rangle_2 = |\lambda\rangle|B_1\rangle \quad (16)$$

For the hypothesized linearity, we would have a generic initial state of $|\psi(0)\rangle$, at time t = 0, resulting from the overlapping of $|\psi(0)\rangle_1$ and $|\psi(0)\rangle_2$:

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\{|\kappa\rangle|B_{0,5}\rangle + |\lambda\rangle|B_{0,5}\rangle\} \quad (17)$$

which evolves, following the measurement done at a time $t > 0$, in the state:

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\{|\kappa\rangle|B_0\rangle + |\lambda\rangle|B_1\rangle\} \quad (18)$$

But this is not true, since it is not possible to verify a contextual overlap of the (macroscopic) measures (simultaneously is marked $|B_0\rangle$ and $|B_1\rangle$). We conclude that, following the measurement of the rating, the state of the micro enterprise will no longer an overlapping of Eigenstates of the measured quantity, but will "collapse" in κ or in λ (or, more correctly, at any point r of the interval [κ, λ]) according to whether the instrument measures 0 or 1 (or an intermediate quantity, to which r, will correspond, of the expected degree of performance)[13].

Now, we consider again (12):

$$\alpha K + \sqrt{1-\alpha^2}e_i < F^{-1}(PD_i)$$

and formula (2):

$$TL_n = \sum_{i=1}^{n} U_i LGD_i EAD_i$$

We defineU_i:

$$U_i = \begin{cases} 1 & if \alpha K + \sqrt{1-\alpha^2}e_i < F^{-1}(PD_i) \\ 0 & otherwise \end{cases} \quad (19)$$

and we have

$$TL_n = \sum_{i=1}^{n} U_i(K, e_i) LGD_i EAD_i \quad (20)$$

In order to maximize $e_i$and K [14], according to the hypothesis thatPD_i, LGD_i and EAD_i are known *a priori*, we consider the economic system as the cohabitation of entities: the banks that need to minimize losses and reduce capital absorption, and the enterprises that need to obtain credit and doing business sustainably and profitably. We remark that *K*, include monetary policy, international economic influences, credibility and trust of the country, etc.

For these reasons we consider a dynamic system in which the bank-enterprise system (and, in turn, families, since we cannot overlook the positive influence, including macroeconomic effects, generated by an efficient banking sector in the transmission of liquidity to the productive complex so that it can remain healthy and performing (and, on the contrasting condition, where there are the negative effects produced by the disappearance of this virtuous cycle) is against micro enterprises (portfolio).

We denote the population of banks with Z_1 and the population of micro enterprises (portfolio) Z_2.

Now, we consider two possible banks strategies:

a) continuing to manage loans using generalist algorithms;
b) reactivating qualitative forms of relationships, trust and credit cooperation.

As exposed by the empirical calculations shown in the section above,in the first case, the result seems to be on the saving on allocations. However, the greater perceived solidity would only be apparent and in the short-term.Both *e* components – with reference to micro and very small enterprises – and *K* are affected, negatively, by the a) strategy. A correlation between the reduction of the productive systemcredit and the higher mortality of the micro enterprises (default of the Z_2 population) and the increase of the non-performing bank loans (default of the Z_1 population) has been observed. Let us introduce the formal statements that support recourse to the b) strategy: whether $z_1 = z_1(t)$ and $z_2 = z_2(t)$ are manifestations of performing loans[15] at time *t* of the two populations Z_1 and Z_2 respectively:

$$\begin{cases} \frac{dz_1}{dt} = z_1 f_1(z_1, z_2) \\ \frac{dz_2}{dt} = z_2 f_2(z_1, z_2) \end{cases} (21)$$

This system of differential equations[16] indicates the dependence of the number of performing loans of each population on that of both; the link between the two populations is mutual.

$$\frac{\partial f_1}{\partial z_2} > 0 \quad \text{and} \quad \frac{\partial f_2}{\partial z_1} > 0 \ (22)$$

A growth in the performance of Z_2 corresponds to a growth in Z_1 (therefore, it is systemic). The mathematical zeros of the model are,

- in the total absence of performance and the consequent extinction of the two populations
- within the limits imposed by the *K* and *e* components, noted upon initial reflection on each enterprise in the Z_2 population, which provide a logistical trend to z_2 with a high point that is always lower than the totality ofZ_2.

High values of z_2 are conditioned by $\alpha K + \sqrt{1-\alpha^2}e_i > F^{-1}(PD_i)$, for which U_i – and therefore TL_n – tends toward zero.

Conclusions

The model's results indicate a need to make a paradigm shift in the evaluation of the creditworthiness of micro and very small enterprises. From the data collected and from the analyses carried out, current methods have proved to be ineffective in adequately supporting the productive system and, in concert with the local and national economic growth. Some points for future research seem to be:

- regulation on the incompatibility of commercial and speculative businesses in the hands of the same bank;
- strengthening studies on qualitative rating systems;
- creation of divisions, inside banks, specialized in the analysis of small economic entities with a relationship banking approach[17];
- consolidation of the cooperative credit system and credit consortiums.

The contribution offers an innovative and rigorous way - also through formalism and conceptualism of quantum mechanics - to the critical study of current rating models and their effects. The model's results indicate a need to make a paradigm shift in the evaluation of the creditworthiness of micro and very small enterprises. Stability, or rather growth, in a real economy founded on micro and very small enterprises, is a function - though not univariate - of the credit leverage disbursed. Less available credit significantly impacts the real economy, especially in countries where the financial system is centred on banks. Clearly, the rating class assigned determines whether a subject can or cannot access credit and how much it will cost. This then orients the performance results for economic micro-entities, which are structurally incapable of replacing financial resources with endogenous means, in the same terms as the Default Probability assigned by the algorithm, thus overturning the principle of cause and effect. The research tries to show how this happens. A review survey was done on national data from Italy with a comparison developed on a local subset from the Sardinia Region, as an empirical demonstration of what has been showed.

Notes

[1] The methodological normalization of the two databases was required, since the data from *Unioncamere* (Chambers of Commerce), collected during the first phase of the project, were in fact redundant. This was due to the fact that the companies, which actually closed, were not extinguished in real time due to a lack of Chamber of Commerce

notification of their cancellation from the Register of Companies. The ISTAT data cited also had not yet been updated to the most recent periods.

[2] The rhythmic change occurring between 2016 and 2017 was mainly due to the sale of bank NPLs to companies specialising in recovery.

[3] Regardless, it is believed that not even an inverse correlation is extant: the macro enterprises appeared to be rather indifferent to the quantitative turbulence of financial leverage, at least over the short-term. Hence, the indicator shows a non-correlation, and obviously cannot be considered reliable in the perception of a negative interdependence.

[4]Cf. Panetta and Signoretti (2010).

[5]Cf. Conti (2016).

[6]Portfolio losses measured *ex post* with minimum offsets.

[7] Cf. Conti (2016).

[8] Cf. Fong and Vasicek (1984).

[9] Cf. Meyn and Tweedie (1993).

[10] The equation can be rewritten: $dS_i = S_i(t)r_i dt + S_i(t)\sigma_i dW_i$. Then $X_i\sqrt{dt} = dW_i$: "Wiener process".

[11] Cf. Desogus and Casu (2018).

[12]The formalism and conceptualism of quantum mechanics are being borrowed for a theoretical analogy.

[13] The empirical demonstration of the above is work in progress. In an independent manner, an experiment was started on a sample of 100 Sardinian micro-enterprises, simulating a reversal of the rating obtained and predicting its effects on the accounting. The first tests carried out so far have shown that the denial of custody to those who have obtained it (and have continued to perform), would have led to signals of default on the 300th day on average. Similarly, a loan to excluded micro enterprises and reaching non-performing enterprises, would have allowed a condition of regularity throughout the first year.

[14] A targeted analysis of the *K* component, based on the same initial postulate, was conducted by Desogus and Casu (2019).

[15] The rationale for default levels could be developed in contrasting terms: the final result would be the same.

[16]Cf. Perko (2001).

[17]Cf. Fernando et al. (2002).

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