Proposing a Model on Preemptive Multi-mode Resource-constrained Project Scheduling Problem

Zahra Zare
Department of Industrial Engineering
Najafabad Branch, Islamic Azad University
Isfahan, Iran.

Abdulreza Naddaf
Department of Management
Shiraz University
Shiraz, Iran.

Mohammad Reza Salehi
Department of Industrial Engineering,
Najafabad Branch, Islamic Azad University
Isfahan, Iran.

Abstract

In this paper, we present a new mathematical model for a preemptive multi-mode resource-constrained project scheduling problem, in which multiple execution modes are available for each activities of the project. Activities are allowed to be preempted at any time and restarted later at no additional cost. The objective is minimization the makespan. We present computational results for the P-MRCPSP which proves validation of proposed model.

Keywords: preemptive, multi mode, resource constrained, makespan

1. Introduction

The resource-constrained project scheduling problem (RCPSP) is to schedule project activities in order to complete a project in the minimum possible time under the presence of precedence and resource constraints. Precedence constraints are defined between activities (i.e., no activity can be started before finishing all its Predecessors). The multi-mode problem (MRCPSP) is a generalized version of the RCPSP, where each activity can be performed in one out of a set of modes, with a specific activity duration and resource requirements. The objective of the MRCPSP is to find a mode and a start time for each activity such that the makespan is minimized and the schedule is feasible with respect to the precedence and resource constraints. In the non preemptive case once started an activity is not interrupted and runs to completion. The preemptive resource-constrained project scheduling problem (PRCPSP) includes the first relaxation and assumes that activities can be pre-empted at any or m integer time instant and restarted later on at no additional cost (Verma, 2006). In the preemptive case which is discussed in this paper an activity can be interrupted any number of times. As this problem is a generalization of the RCPSP, the P-MRCPSP is also NP-hard (Alcaraz et al., 2003). The organization of this paper is as follows: Section 2 focuses on the literature review about resource-constrained project scheduling problems. In Section 3 we first describe the problem and then propose its mathematical model. Numerical example is given in section 4 and Finally, conclusion is presented in Section 5.

2. Literature review

The basic RCPSP assumes that an activity cannot be interrupted once it has been started. Bianco et al. (1999), Brucker and Knust (2001), Debels and Vanhoucke (2008), Demeulemeester and Herroelen (1996) and Nudtasomboon and Randhawa (1997) allow activity preemption at discrete points in time, that is, an activity can be interrupted after each integer unit of its processing time. e. Franck et al. (2001) propose a calendar concept for project scheduling which includes preemptive scheduling. A calendar is defined as a binary function that determines for each period whether activity execution is possible or a break occurs during which an activity may not be started or continued.

Alcaraz et al. (2003), Bouleimen and Lecocq (2003), Hartmann (2001), Jarboui et al. (2008), Jozefowska et al. (2001), Ozdamar (1999) and Pesch (1999) while Varma et al. (2007) discuss a multi-mode problem without nonrenewable resources. Multi-mode problems with generalized precedence constraints have been considered by Barrios et al. (2011), Brucker and Knust (2007), Calhoun et al. (2002), Reyck and Herroelen (1999), Drexl et al. (2000), Heilmann (2001-2003), Nonobe and Ibaraki (2002), and Sabzehparvar and Seyed Hosseini (2008). Zhu et al. (2006) employ a multi-mode problem with generalized resource constraints. Salewski et al. (1997) and Drexl et al. (2000) extend the multi-mode RCPSP by introducing socalled mode identity constraints. The motivation for this is that there may be several activities that should be performed in the same way, e.g., by allocating the same resources to them. To cover this, the set of all activities is partitioned into sets of activities H_u, $u = 1,...,U$. The activities of each set *Hu* must be performed in the same mode. That is, $M_i = M_j$ is must hold for all activities $i, j \in H_u$ (note that this requires $M_i = M_j$). Schultmann and Rentz (2001) present a case study that demonstrates how the multi-mode RCPSP can be applied to projects which consist of the dismantling of buildings.

Voß and Witt (2007) employ the multi-mode RCPSP with an objective that contains makespan, weighted tardiness and setup costs. The inclusion of setup costs supports batching of activities. The problem setting is motivated by a production planning problem at a steel manufacturer. Bomsdorf and Derigs (2008) employ an objective for movie shooting projects that consists of several components which are allowed to be squared. The components include specific criteria such as the minimization of location changes over time (each activity is associated with a location). Al-Fawzan and Haouari (2005) combine makespan minimization and maximization of total free slack into one objective. Another way to deal with multiple objectives is the generation of Pareto-optimal schedules. This approach is followed by several authors. Davis et al. (1992) minimize the makespan as well as the overutilization of each renewable resource.

3. Proposed model

3.1 Problem definition

The project is represented as an activity-on-the-node network $G(N, A)$, where N is the set of activities and A is the set of pairs of activities between which a finish-start precedence relationship with a minimal time lag of 0 exists. A set of activities, numbered from 1 to $|N|$ with a dummy start node 0 and a dummy end node $|N| + 1$, is to be scheduled on a set R resource. Each activity $i \in N$ is performed in a mode m_i, which is chosen out of a set of $|M_i|$ different execution modes $M_i=\{1,...,. |M_i|\}$. The duration of activity i, when executed in mode m_i, is d_{im_i}. Each mode m_i requires $r_{im_i,k}$ nonrenewable resource units *and* $r_{im,z}$ renewable resource units. A schedule S is defined by a vector of activity start times s_i and a vector denoting its corresponding execution modes m_i. A schedule is said to be feasible if all precedence and resource constraints are satisfied. The objective of the P-MRCPSP is to minimize the makespan of the project. In the P-MRCPSP, activities are allowed to be preempted at any time and restarted later on at no additional cost. Therefore, each duration unit v of an activity i scheduled in mode m_i (with $v \in \{0, ..., d_{im_i} - 1\}$) is assigned a starting time s_{iv}. The objective is minimization the makespan. The makespan is the completion time of a project that equals the completion time of activity n

3.2 Mathematical model

In this section, we present a novel mathematical model for a preemptive multi-mode resource constrained project scheduling problem (P-MRCPSP).

3.2.1 Indices and parameters and Variables

T	project time window
N	number of activity
i	index of activity
0	dummy start node
$n+1$	dummy end node
m	Index of mode
$S_{i,lm}$	start time of l^{th} units of activity i in mode m where each activity i is broken in to d_{im}
d_{im}	duration of Activity i executed in mode m

t	index for period of time
k	index of nonrenewable resource
z	index of renewable resource
a_k	availability of each nonrenewable resource type k in each time period
a_Z	availability of each renewable resource type z in each time period
$r_{im,k}$	each activity i in mode m requires $r_{im,k}$ nonrenewable resource units
$r_{im,z}$	each activity i in mode m requires $r_{im,z}$ renewable resource units
x_{im}	1 if activity i is completed in mode m; 0 otherwise
y_{iht}	1 if h^{th} units for activity i is executed in time priod t; 0 otherwise
E	a very large positive number

3.2.2 Proposed mathematical model

The P-MRCPSP can be stated as follows:

$$\min \ Makespan \quad \min\left(S_{n+1,0}\right) \tag{1}$$

S.T.

$$S_{0,0} = 0 \tag{2}$$

$$S_{i,d_{im}} + 1 \le S_{j,0m} + (1 - x_{im})\mathrm{E} + (1 - x_{jm})\mathrm{E} \qquad m = 1,..M, \qquad i = 1,\ldots,N \tag{3}$$

$$S_{i,(v-1)m} + 1 \le S_{i,vm} + (1 - x_{im})\mathrm{E} \qquad m = 1,..M, \qquad i = 1,\ldots,N \tag{4}$$

$$S_{n+1,0} \le T \tag{5}$$

$$\sum_{i=1}^{N}\sum_{m=1}^{M} x_{im} = 1 \tag{6}$$

$$\sum_{i=1}^{N}\sum_{t=0}^{T} y_{iht} = 1 \tag{7}$$

$$\sum_{i=1}^{N}\sum_{t=0}^{T} (t \times y_{iht}) - \sum_{m=1}^{M} S_{ijm} = 0 \tag{8}$$

$$\sum_{i=1}^{N}\sum_{m=1}^{M} r_{im_i,k} x_{im} \le a_k \tag{9}$$

$$\sum_{t=0}^{T}\sum_{i=1}^{N} y_{iht} \times \sum_{m=1}^{M} (x_{ijm} \times r_{imz}) \le a_z \tag{10}$$

$$S_{i,vm} \in \mathrm{int}^{+} \tag{11}$$

The objective function (1) minimizes the total makespan of the project. Constraint (2) forces the project to start at time instance zero Constraint set (3), the earliest start time of an activity j cannot be smaller than the finish time for the last unit of duration of its predecessor i. Constraint set (4) guarantees that the start time for every time instance of an activity has to be at least one time-unit larger than the start time for the previous unit of duration and Constraint set (5) makes the makespan not to take more than T. Each activity has to be performed in exactly one mode m (constraint (6)). Constraint (7-8) is used to determine the execution time period of each activity sections. Constraint (9-10) takes care of the nonrenewable and renewable resource limitations, respectively and constraint (11) ensures that the activities start times assume nonnegative integer values. A schedule which fulfills all the constraints (1-11) is called optimal.

4. Experimental Results

Validation of mathematical modeling is proven by solving a small-size problem of P-MRCPSP. For this reason, a problem is chosen consisting of 6 activities, in which two activities (1,6) are dummy. The information of project are shown in table1 for m=1 and in table 2 for m=2 and the final results are shown in table 3 to table 5.

5. Conclusion

The pervious researches have shown that activity preemption drastically increases the problem complexity. In this paper, we have introduced a novel mathematical model for the preemptive multi-objective multi-mode resource constrained project scheduling problem (P-MRCPSP). In fact we have extended the resource constrained project schedule problems by considering preemption. The objective was minimization the makespan and finally we solved a small-size problem of P-MRCPSP to prove the validating the proposed model. Areas of further researches are to propose other models that consider new constraints for RCPSP. Clearly for solving this kind of problems in large size we need to use metaheuristic algorithms.

References

Alcaraz, J., Maroto, C., & Ruiz, R. (2003). Solving the multi-mode resource constrained project scheduling problem with genetic algorithms. Journal of the Operational Research Society, 54(6), 614-626.

Al-Fawzan, M. A., & Haouari, M. (2005). A bi-objective model for robust resource-constrained project scheduling. International Journal of Production Economics, 96(2), 175-187.

Barrios, A., Ballestin, F., & Valls, V. (2011). A double genetic algorithm for the MRCPSP/max. Computers & Operations Research, 38(1), 33-43.

Bianco, L., Caramia, M., & DellOlmo, P. (1999). Solving coloring techniques. In Weglarz (1999), a preemptive project scheduling problem with pages 135– 146.

Bomsdorf, F., & Derigs, U. (2008). A model, heuristic procedure and decision support system for solving the movie shoot scheduling problem. Or Spectrum, 30(4), 751-772.

Bouleimen, K., & Lecocq, H. (2003). A new efficient simulated annealing algorithm for the resource constrained project scheduling problem and its multiple mode versions. European Journal of Operational Research, 142(2), 268-281.

Brucker, P., & Knust, S. (2001). Resource-constrained project scheduling and timetabling. Lecture Notes in Computer Science, Vol. 2079, 277-293.

Calhoun, K. M., Deckro, R. F., Moore, J. T., Chrissis, J. W., & Hove, J. C. V. (2002). Planning and replanning in project and production scheduling. Omega – The international Journal of Management Science, 30(3), 155-170.

Davis, K. R., Stam, A., & Grzybowski, R. A. (1992). Resource constrained project scheduling with multiple objectives: A decision support approach. Computers & Operations Research, 19(7), 657-669.

Debels, D., & Vanhoucke, M. (2008). The impact of various activity assumptions on the lead time and resource utilization of resource-constrained projects. Computers & Industrial Engineering, 54(1), 140-154.

Demeulemeester, E. L., & Herroelen, W. S. (1996). An efficient optimal solution procedure for the preemptive resource-constrained project scheduling problem. European Journal of Operational Research, 90(2), 334-348.

Drexl, A., Nissen, R., Patterson, J. H., & Salewski, F. (2000). Progen/px - an instance generator for resource-constrained project scheduling problems with partially renewable resources and further extensions. European Journal of Operational Research, 125(1), 59-72.

Franck, B., Neumann, K., & Schwindt, C. (2001). Project scheduling with calendars. Or Spektrum, 23, 325- 334.

Hartmann, S. (2001). Project scheduling with multiple modes: A genetic algorithm. Annals of Operations Research, 102 (1-4), 111-135.

Heilmann, R. (2001). Resource-constrained project scheduling: a heuristic for the multi-mode case. Or Spektrum, 23(3), 335-357.

Heilmann, R. (2003). A branch-and-bound procedure for the multi-mode resource-constrained project scheduling problem with minimum and maximum time lags. European Journal of Operational Research, 144(2), 348-365.

Jarboui, B., Damak, N., Siarry, P., & Rebai, A. (2008). A combinatorial particle swarm optimization for solving multi-mode resource-constrained project scheduling problems. Applied Mathematics and Computation, 195(1), 299-308.

Jozefowska, J., Mika, M., Rozycki, R., Waligora, G., & Weglarz, J. (2001). Simulated annealing for multimode resource-constrained project scheduling. Annals of Operations Research, 102(1-4), 137-155.

Nonobe, K., & Ibaraki, T. (2002). Formulation and tabu search algorithm for the resource constrained project scheduling problem. In C. C. Ribeiro and P. Hansen, editors, Essays and Surveys in Metaheuristics, pages 557–588. Kluwer Academic Publishers.

Nudtasomboon, N., & Randhawa, S. U. (1997). Resource constrained project scheduling with renewable and non-renewable resources and time-resource tradeoffs. Computers & Industrial Engineering, 32(1), 227-242.

Ozdamar, L. (1999). A genetic algorithm approach to a general category project scheduling problem. IEEE Transactions on Systems, Man, and Cybernetics, Part C: Applications and Reviews, 29(1), 44-59.

Pesch, E. (1999). Lower bounds in different problem classes of project schedules with resource constraints. In Weglarz (1999), pages 53–76.

Reyck, B., & Herroelen, W. S. (1999). The multi-mode resource-constrained project scheduling problem with generalized precedence relations. European Journal of Operational Research, 119(2), 538-556.

Sabzehparvar, M., & Seyed-Hosseini, S. M. (2008). A mathematical model for the multi-mode resource constrained project scheduling problem with mode dependent time lags. Journal of Supercomputing, 44(3), 257-273.

Salewski, F., Schirmer, A., & Drexl, A. (1997). Project scheduling under resource and mode identity constraints: Model, complexity, methods, and application. European Journal of Operational Research, 102(1), 88-110.

Schultmann, F., and Rentz, O. (2001). Environment-oriented project scheduling for the dismantling of buildings. Or Spectrum, 23(1), 51-78.

Varma, V. A., Uzsoy, R., Pekny, J., & Blau, G. (2007). Lagrangian heuristics for scheduling new product development projects in the pharmaceutical industry. Journal of Heuristics, 13(5), 403-433.

Verma, S. (2006). Exact methods for the preemptive resource constrained project scheduling problem. research and publication, pp.1-17.

Voß, S., & Witt, A. (2007). Hybrid flow shop scheduling as a multi-mode multi-project scheduling problem with batching requirements: A real-world application. International Journal of Production Economics, 105(2), 445-458.

Weglarz, J. (1999). Project scheduling: Recent models, algorithms and applications. Kluwer Academic Publishers.

Zhu, G.., Bard, J. F., & Yu, G. (2006). A branch-and-cut procedure for the multimode resource-constrained project-scheduling problem. INFORMS Journal on Computing, 18(3), 377-390.

Table 1.information of project for m=1

activities	d_i	$r_{i1,k}$	$r_{i1,z}$	successor
0	0	0	0	1,2
1	3	3	3	4,5
2	4	4	5	3,5
3	1	7	5	5
4	2	1	7	5
5	0	0	0	-

Table 2.information of project for m=2

activities	d_i	$r_{i2,k}$	$r_{i2,z}$	successor
0	0	0	0	1,2
1	2	5	4	4,5
2	3	2	4	3,5
3	2	4	4	5
4	1	3	2	5
5	0	0	0	-

a_k=10
a_z=10
T=6

Table3. Computational results

x_{11}	x_{12}	x_{21}	x_{22}	x_{31}	X_{32}	x_{41}	x_{42}	Min makespan
1	**0**	**0**	**1**	**0**	**1**	**1**	**0**	**5**

Table4. Computational results

$s_{1,11}$	$s_{1,21}$	$s_{1,31}$	$s_{2,11}$	$s_{2,21}$	$s_{2,31}$	$s_{2,41}$	$s_{3,11}$	$s_{3,21}$
0	**1**	**2**	**0**	**0**	**0**	**0**	**0**	**0**

Table5. Computational results

$s_{4,11}$	$s_{1,12}$	$s_{1,22}$	$s_{2,12}$	$s_{2,22}$	$s_{2,32}$	$s_{3,12}$	$s_{4,12}$	$s_{4,22}$	$s_{5,0}$
4	**0**	**0**	**0**	**0**	**1**	**2**	**3**	**0**	**0**