The Timing of Team Production Incorporating Reciprocity

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Abstract

This paper develops a game model of team production incorporating the preference of reciprocity to probe the intrinsic mechanism how the reciprocity influences the team efficiency under different game timing, which is compared with that of inequity aversion and selfish preference respectively. It is found that the reciprocity may promote the Pareto improvement of the team efficiency under looser condition than that of inequity aversion. As long as the final mover is reciprocal instead of selfish, the extent of Pareto improvement under sequential game is bigger than that under simultaneous game even if other agents except the final one move randomly. Therefore, the principal should screen and select reciprocal agents to establish team and ensure that they move sequentially. It offers a new theoretical explanation for the existence of the principal, and a new approach for team incentives.

Key Words: Reciprocity, Team production, Game timing, Incentive mechanism

1. Introduction

As a typical multi-agent problem, team production was studied extensively in economics and game theory. It was difficult to distinguish and measure the contribution of each individual agent, which resulted in the free-riding problem among agents and the team production inefficiencies, far below the Pareto optimal level of team cooperation, as illustrated by Alchian and Demsetz (1972). A variety of approaches such as the principal of breaking the balanced budget of Holmstrom (1982), the incentive contracts with proper structure of McAfee and McMillan (1991) and the peer sanction in long-term repeated relationships of Che and Yoo (2001), could improve the production efficiency and promote team cooperation to some extent. In recent years, with the development of behavioral economics and behavioral game theory, it was found that psychological preferences such as inequity aversion and reciprocity, which were neglected by the traditional economics, could promote team cooperation to some extent automatically.

Based on the Fehr-Schmidt model emphasizing inequity aversion, which refers to the behaviors of sacrificing own material payoff to improve fairness of income distribution, Wei and Qin (2008) found that the sufficiently strong inequity aversion could achieve team cooperation. Rey Biel (2008) proven that inequity aversion was an internal factor to promote team formation autonomously, and achieved team cooperation by creating inequity off equilibrium, by which agents feel envious or guilt. Li (2009) found that team cooperation could be sustained through a balanced budget if inequity aversion is sufficient strong, which punished some agents randomly. Li (2009) pointed out that it was possible for inequity aversion to enhance and induce the team efficiency, and even to achieve the Pareto optimality under some specific conditions. Bartling and Siemens (2010) proven that the condition under which inequity aversion could realize team cooperation became less restrictive as the scale of the team became smaller. Bartling (2011) drew the conclusion that inequity aversion would increase the agent's cost when his payoff was lower than those of peers, and thereby may limit the realization of team cooperation.

Based on the Rabin model emphasizing reciprocity, which refers to the behaviors of sacrificing own material payoff to repay those who are kind and to revenge those who are bad to him in Rabin (1993), Wu et al. (2010) found that reciprocity could promote the Pareto improvement of team production, and even could achieve the Pareto optimality with the specific reciprocity intensity and team scale. Qian and Pu (2011) indicated that the impact of reciprocity on the team efficiency under different conditions varied widely, may enhance or decrease the team efficiency, where the belief about the behavioral intention of peers was a very important factor.

These analyzed the influence of inequity aversion and reciprocity on the team efficiency, but they were in the framework of simultaneous game without considering the game timing of agents. Particularly, by comparing the team efficiency under different game timing based on the Fehr-Schmidt model, Huck and Rey Biel (2006) found there were conformity effect of inequity aversion in the simultaneous game and commitment effect of inequity aversion in the sequential game, which both could improve the team efficiency, and the degree of improvement in the sequential game was greater. However, the study about the inherent mechanism how reciprocity affected the team efficiency at different game timing has not been found yet.

In fact, reciprocity emphasizes fairness of the behavioral intention and inequity aversion emphasizes fairness of the behavioral outcome, which both are important factors in behavioral choice decisions. But, in the sequential game the role of reciprocity may be more obvious, because a lot of experiments and empirical observations have shown that the intention of leading mover affects the behavioral choice of the following mover directly in the sequential game, as exhibited by Wei and Li (2013). Consequently, based on the Rabin model, by exploring and comparing the team efficiency under different game timing, this paper will analyze the inherent mechanism how reciprocity affects the team efficiency at different game timing, and compare it with that of inequity aversion and selfish preference respectively. It offers a new theoretical explanation for the existence of the principal, and a new approach for team incentives.

2. Selfish Team Model

For simplicity and generality taken widely in literatures such as Huck and Rey Biel (2006), we assume that a team includes two risk neutral agents, each agent i ($i = 1, 2$) chooses some effort $x_i \geq 0$, which is unobservable by the principal. Let the output of team y be linear in efforts x_i, then

$$y = 2(k_1 x_1 + k_2 x_2) \tag{1}$$

where $k_i \geq 0$ denotes the productivity of agent i, and the greater the value, the stronger the productivity.

Since the principal cannot observe the agents' efforts, and hereby cannot determine the contribution of each agent to the team output, according to the prevailing practices in the literatures, the team output is assumed to be distributed equally among agents. Thus, the material payoff of agent i is

$$m_i(x_i \mid x_j) = \frac{1}{2} y = k_i x_i + k_j x_j \tag{2}$$

where $j = 1, 2$ and $i \neq j$.

Furthermore, denote the effort cost function of agent i as

$$c_i(x_i) = \frac{1}{2} x_i^2 \tag{3}$$

Then, according to (2) and (3), the utility of agent i who is risk neutral is

$$\pi_i(x_i \mid x_j) = k_i x_i + k_j x_j - \frac{1}{2} x_i^2 \tag{4}$$

Agent i pursuits the maximum utility π_i by selecting the appropriate efforts x_i, then

$$x_i^{self} = \arg\max_{x_i} \pi_i(x_i \mid x_j) = k_i x_i + k_j x_j - \frac{1}{2} x_i^2 \tag{5}$$

which represents the optimal effort of agent i when he is purely selfish and hereby only cares about his own material payoff. Then,

$$x_i^{self} = k_i \tag{6}$$

By (1) and (6), if agents are selfish, the team output is

$$y^{self} = 2(k_i^2 + k_j^2) \tag{7}$$

3. Utility Function of Reciprocity

If an agent is reciprocal, he will repay the action of kind intention and revenge the action of bad intention, although it is possible to sacrifice some his own material payoffs. According to the above conditions and the Rabin model [12], after incorporating reciprocity, the utility function of agent i is

$$u_i(x_i \mid x_j) = m_i(x_i \mid x_j) - c_i(x_i) + \gamma_i \tilde{f}_j(1+f_i) \tag{8}$$

where $m_i(x_i \mid x_j)$ represents the acquired material payoff, $c_i(x_i)$ represents the effort cost, and $\gamma_i \tilde{f}_j(1+f_i)$ represents the psychological utility of reciprocity.

Each parts of the psychological utility of reciprocity $\gamma_i \tilde{f}_j(1+f_i)$ are explained as follows.

Firstly, γ_i means the coefficient of measuring the reciprocity intensity of agent i. The coefficient did not exist in the original Rabin model, while the literature Wu et al. (2010) also introduced. The reciprocity intensity coefficient can characterize the degree how the agent concerns the reciprocal psychological utility, and also can facilitate the analysis of the effect of the reciprocity intensity on the team efficiency. Particularly, if $\gamma_i = 0$, which means that the reciprocity intensity is 0, the utility function of the above (8) degenerates to (4) which describes the case of selfish preference.

Secondly, f_i means the kindness of agent i's behavior to agent j. If $f_i > 0$, agent i's behavioral intention is kind. If $f_i < 0$, agent i's behavioral intention is bad. If $f_i = 0$, agent i's behavioral intention is neutral. A greater absolute value of f_i indicates a stronger kindness of the behavioral intention. Rabin defined f_i as $f_i = \frac{m_j^s - m_j^e}{m_j^h - m_j^l}$, where m_j^s denotes the material payoff that agent j actually gets, m_j^h represents the highest material payoff that agent j can obtain under the given conditions, m_j^l represents the lowest material payoff that agent j can obtain under the given conditions, m_j^e represents the equitable material payoff that agent j should receive , which equals the mean of m_j^h and m_j^l. Under the above backgrounds, we get $m_j^s = k_i x_i + k_j x_j$, $m_j^h = 2(k_i^2 + k_j^2)$, $m_j^l = 0$ and $m_j^e = k_i^2 + k_j^2$. Thus,

$$f_i = \frac{m_j^s - m_j^e}{m_j^h - m_j^l} = \frac{(k_i x_i + k_j x_j) - (k_i^2 + k_j^2)}{2(k_i^2 + k_j^2) - 0} = \frac{k_i x_i + k_j x_j}{2(k_i^2 + k_j^2)} - \frac{1}{2} \tag{9}$$

Finally, $\tilde{f}_j$ means the agent i's belief about the kindness of agent j's behavior to agent i. If $\tilde{f}_j > 0$, agent i believes the behavioral intention of agent j to agent i is kind. If $\tilde{f}_j < 0$, agent i believes the behavioral intention of agent j to agent i is bad. If $\tilde{f}_j = 0$, agent i believes the behavioral intention of agent j to agent i is neutral. The greater the absolute value of $\tilde{f}_j$, the stronger kindness that agent i believes the behavioral intention of agent j to agent i. According to Rabin (1993), $\tilde{f}_j = \frac{m_i^s - m_i^e}{m_i^h - m_i^l}$. Under the above conditions, we can find $m_i^s = k_i x_i + k_j x_j$, $m_i^h = 2(k_i^2 + k_j^2)$, $m_i^l = 0$ and $m_i^e = k_i^2 + k_j^2$. Then

$$\tilde{f}_j = \frac{m_i^s - m_i^e}{m_i^h - m_i^l} = \frac{k_i x_i + k_j x_j}{2(k_i^2 + k_j^2)} - \frac{1}{2} \tag{10}$$

Substituting (2), (3), (9) and (10) into (8), the utility of agent i is incorporated reciprocity as

$$u_i(x_i \mid x_j) = k_i x_i + k_j x_j - \frac{1}{2}x_i^2 + \gamma_i\left[\frac{k_i x_i + k_j x_j}{2(k_i^2 + k_j^2)} - \frac{1}{2}\right]\left[\frac{k_i x_i + k_j x_j}{2(k_i^2 + k_j^2)} + \frac{1}{2}\right] \tag{11}$$

4. Simultaneous Game

4.1 Equilibrium Efforts

In the simultaneous game, agent i and j choose efforts simultaneously, which decides the team output jointly.

Agent i pursuits the maximum utility $u_i(x_i \mid x_j)$ by selecting the optimal efforts x_i. In (11), from the first-order conditions with respect to x_i, we get

$$\frac{\partial u_i}{\partial x_i} = k_i - x_i + \gamma_i \frac{k_i(k_i x_i + k_j x_j)}{2(k_i^2 + k_j^2)^2} = 0 \qquad (12)$$

So, the reaction function of agent i is

$$x_i^*(x_j) = \frac{2k_i(k_i^2 + k_j^2)^2 + k_i k_j \gamma_i x_j}{2(k_i^2 + k_j^2)^2 - k_i^2 \gamma_i} \qquad (13)$$

It can clearly be seen that the effort choices of reciprocal agents are strategically complementary. If $\gamma_i = 0$, which means the agent is purely selfish, the above equation reduces to equation (6), by which the effort choices of selfish agents is strategically independent. Similarly, agent j's reaction function is

$$x_j^*(x_i) = \frac{2k_j(k_i^2 + k_j^2)^2 + k_i k_j \gamma_j x_i}{2(k_i^2 + k_j^2)^2 - k_j^2 \gamma_j} \qquad (14)$$

In (13) and (14), because of $x_i > 0$, we can find $2(k_i^2 + k_j^2)^2 - k_i^2 \gamma_i > 0$ and $2(k_i^2 + k_j^2)^2 - k_j^2 \gamma_j > 0$.

According to (13) and (14), the equilibrium efforts of agent i and j, x_i^{SIM} and x_j^{SIM}, are

$$x_i^{SIM} = \frac{k_i[2(k_i^2 + k_j^2)^2 + k_j^2(\gamma_i - \gamma_j)]}{2(k_i^2 + k_j^2)^2 - (k_i^2 \gamma_i + k_j^2 \gamma_j)} \qquad (15)$$

and

$$x_j^{SIM} = \frac{k_j[2(k_i^2 + k_j^2)^2 + k_i^2(\gamma_j - \gamma_i)]}{2(k_i^2 + k_j^2)^2 - (k_i^2 \gamma_i + k_j^2 \gamma_j)} \qquad (16)$$

In (15) and (16), due to $x_i > 0$, $2(k_i^2 + k_j^2)^2 - k_i^2 \gamma_i > 0$ and $2(k_i^2 + k_j^2)^2 - k_j^2 \gamma_j > 0$, it is easy to find that $2(k_i^2 + k_j^2)^2 - (k_i^2 \gamma_i + k_j^2 \gamma_j) > 0$.

In (15), from the partial derivative of agent i's equilibrium efforts with respect to the reciprocity intensity γ_i of agent i and the reciprocity intensity γ_j of agent j,

$$\frac{\partial x_i^{SIM}}{\partial \gamma_i} = \frac{k_i(k_i^2 + k_j^2)[2(k_i^2 + k_j^2)^2 - k_j^2 \gamma_j]}{[2(k_i^2 + k_j^2)^2 - (k_i^2 \gamma_i + k_j^2 \gamma_j)]^2} > 0 \qquad (17)$$

and

$$\frac{\partial x_i^{SIM}}{\partial \gamma_j} = \frac{k_i k_j^2 (k_i^2 + k_j^2)^2 \gamma_i}{[2(k_i^2 + k_j^2)^2 - (k_i^2 \gamma_i + k_j^2 \gamma_j)]^2} > 0 \qquad (18)$$

Similarly, in (16), $\frac{\partial x_j^{SIM}}{\partial \gamma_j} > 0$ and $\frac{\partial x_j^{SIM}}{\partial \gamma_i} > 0$.

Then, the following conclusion can be drawn. Firstly, the equilibrium effort increase with own reciprocity. The effort of a reciprocal agent must be higher than that of selfish. A stronger reciprocity will result in a higher effort. Secondly, the equilibrium efforts also increases with peers' reciprocity. Reciprocity will promote agents to adjust efforts according to those of peers, and the adjustment depends on the reciprocity intensity of his own and peers. Peers' reciprocity will promote the agent to choose a higher effort, and his own reciprocity will promote the peers to choose higher efforts. Facing the high efforts of peers, a reciprocal agent will in return choose a high effort too. Meanwhile, when facing reciprocal peers, choosing a high effort can get high efforts of peers in return.This is a circular process. Based on the above two aspects, reciprocity does promote team cooperation and hereby promote the Pareto improvement of the team production efficiency.

4.2 Equilibrium Output

According to (1), (15) and (16), the team output is

$$y^{SIM} = 2(k_i x_i^{SIM} + k_j x_j^{SIM}) = \frac{2(k_i^2 + k_j^2)}{1 - \frac{k_i^2\gamma_i + k_j^2\gamma_j}{2(k_i^2 + k_j^2)^2}} \tag{19}$$

From $2(k_i^2 + k_j^2)^2 - (k_i^2\gamma_i + k_j^2\gamma_j) > 0$, we find that $0 < \frac{k_i^2\gamma_i + k_j^2\gamma_j}{2(k_i^2 + k_j^2)^2} < 1$, and hereby $0 < 1 - \frac{(k_i^2\gamma_i + k_j^2\gamma_j)}{2(k_i^2 + k_j^2)^2} < 1$. Then, from (7) and (19), we get $y^{SIM} > y^{self}$, by which reciprocity enhances the team output.

Furthermore, in (19), from the partial derivative of the equilibrium team output with respect to the reciprocity intensity γ_i of agent i and the reciprocity intensity γ_j of agent j,

$$\frac{\partial y^{SIM}}{\partial \gamma_i} = \frac{4k_i^2(k_i^2 + k_j^2)^3}{[2(k_i^2 + k_j^2)^2 - (k_i^2\gamma_i + k_j^2\gamma_j)]^2} > 0 \tag{20}$$

and

$$\frac{\partial y^{SIM}}{\partial \gamma_j} = \frac{4k_j^2(k_i^2 + k_j^2)^3}{[2(k_i^2 + k_j^2)^2 - (k_i^2\gamma_i + k_j^2\gamma_j)]^2} > 0 \tag{21}$$

Clearly, the team output increases with the reciprocity. In the team containing reciprocators, the equilibrium output is higher than that of the team only containing purely selfish agents. Whenever at least one agent is reciprocal agent in the team, the team production efficiency will achieve the Pareto improvement to some extent. The stronger the reciprocity intensity is, the greater the degree of the Pareto improvement is.

Summarily, based on the above equilibrium efforts and team output, comparing with that of selfish preference, the reciprocity promotes the Pareto improvement of team production. Therefore, the principal should identify the preference type of agents, and select reciprocal employees to form team, because they will choose higher efforts and will motivate others to choose higher efforts too.

5. Sequential Game

5.1 Equilibrium Efforts

In the sequential game, agents choose their efforts sequentially, and the following mover knows the effort choice of the leading mover. Notationwise, let agent i be the leading mover and agent j be the following mover.

By backwards induction, the following mover agent j has to choose his effort according to (14) after seeing the effort x_i chosen by leading mover agent i. Anticipating this, the leading mover will maximize his utility $u_i(x_i)$. So, putting (14) into (11), we obtain

$$u_i(x_i) = k_i x_i - \frac{1}{2}x_i^2 - \frac{1}{4}\gamma_i + k_j \frac{2k_j(k_i^2 + k_j^2)^2 + k_i k_j \gamma_j x_i}{2(k_i^2 + k_j^2)^2 - k_j^2\gamma_j} + \gamma_i \frac{[k_i x_i + k_j \frac{2k_j(k_i^2 + k_j^2)^2 + k_i k_j \gamma_j x_i}{2(k_i^2 + k_j^2)^2 - k_j^2\gamma_j}]^2}{4(k_i^2 + k_j^2)^2} \tag{22}$$

By the first-order condition, the equilibrium effort x_i^{SEQ} of the leading mover agent i is

$$x_i^{SEQ} = \frac{2k_i(k_i^2 + k_j^2)^2[2(k_i^2 + k_j^2)^2 + k_j^2(\gamma_i - \gamma_j)]}{[2(k_i^2 + k_j^2)^2 - k_j^2\gamma_j]^2 - 2\gamma_i k_i^2(k_i^2 + k_j^2)^2} \tag{23}$$

Substituting it into (14), the equilibrium effort x_j^{SEQ} of the following mover agent j is

$$x_j^{SEQ} = \frac{2k_j(k_i^2 + k_j^2)^2[2(k_i^2 + k_j^2)^2 + k_i^2(\gamma_j - \gamma_i) - k_j^2\gamma_j]}{[2(k_i^2 + k_j^2)^2 - k_j^2\gamma_j]^2 - 2\gamma_i k_i^2(k_i^2 + k_j^2)^2} \tag{24}$$

In (23) and (24), from the partial derivative of the equilibrium efforts with respect to the reciprocity intensity, we can get

$$\frac{\partial x_i^{SEQ}}{\partial \gamma_i} = \frac{2k_i(k_i^2 + k_j^2)^2[2(k_i^2 + k_j^2)^2 - \gamma_j k_j^2][2(k_i^2 + k_j^2)^3 - \gamma_j k_j^4]}{\{[2(k_i^2 + k_j^2)^2 - \gamma_j k_j^2] - 2\gamma_i k_i^2(k_i^2 + k_j^2)^2\}^2} \tag{25}$$

$$\frac{\partial x_i^{SEQ}}{\partial \gamma_j}=\frac{2k_ik_j^2(k_i^2+k_j^2)^2\{[2(k_i^2+k_j^2)^2-\gamma_jk_j^2][2(k_i^2+k_j^2)^2-\gamma_jk_j^2+2\gamma_ik_j^2]+2\gamma_ik_i^2(k_i^2+k_j^2)^2\}}{\{[2(k_i^2+k_j^2)^2-\gamma_jk_j^2]-2\gamma_ik_i^2(k_i^2+k_j^2)^2\}^2} \quad (26)$$

$$\frac{\partial x_j^{SEQ}}{\partial \gamma_i}=\frac{2\gamma_jk_jk_i^2(k_i^2+k_j^2)^2[2(k_i^2+k_j^2)^3-\gamma_jk_j^4]}{\{[2(k_i^2+k_j^2)^2-\gamma_jk_j^2]-2\gamma_ik_i^2(k_i^2+k_j^2)^2\}^2} \quad (27)$$

and

$$\frac{\partial x_j^{SEQ}}{\partial \gamma_j}=\{[2(k_i^2+k_j^2)^3-\gamma_jk_j^4][2(k_i^2+k_j^2)^2-\gamma_ik_i^2-\gamma_jk_j^2]+\gamma_jk_i^2k_j^2[2(k_i^2+k_j^2)^2-\gamma_jk_j^2]+\gamma_i\gamma_jk_i^2k_j^2\}$$

$$\times\frac{2k_j(k_i^2+k_j^2)^2}{\{[2(k_i^2+k_j^2)^2-\gamma_jk_j^2]-2\gamma_ik_i^2(k_i^2+k_j^2)^2\}^2} \quad (28)$$

Because of $2(k_i^2+k_j^2)^2-k_i^2\gamma_i>0$ and $2(k_i^2+k_j^2)^2-k_j^2\gamma_j>0$, and hereby $2(k_i^2+k_j^2)^2-(k_i^2\gamma_i+k_j^2\gamma_j)>0$ and $2(k_i^2+k_j^2)^3-\gamma_jk_j^4=2k_i^2(k_i^2+k_j^2)^2+k_j^2[2(k_i^2+k_j^2)^2-\gamma_jk_j^2]>0$, in (25) to (28), we can find that there must be $\frac{\partial x_i^{SEQ}}{\partial \gamma_i}>0$, $\frac{\partial x_i^{SEQ}}{\partial \gamma_j}>0$, $\frac{\partial x_j^{SEQ}}{\partial \gamma_j}>0$ and $\frac{\partial x_j^{SEQ}}{\partial \gamma_i}>0$.

Thus, in the sequential game, reciprocity will also enhance the efforts, including those of the leading and the following agents. Moreover, similar with that of simultaneous game, the equilibrium efforts also increase with both the reciprocity intensity of own and peers.

5.2 Equilibrium Output

According to the equilibrium efforts (23) and (24) and the team production function (1), the equilibrium team output y^{SEQ} is

$$y^{SEQ}=2(k_ix_i^{SEQ}+k_jx_j^{SEQ})=\frac{4(k_i^2+k_j^2)^2[2(k_i^2+k_j^2)^3-k_j^4\gamma_j]}{[2(k_i^2+k_j^2)^2-k_j^2\gamma_j]^2-2\gamma_ik_i^2(k_i^2+k_j^2)^2} \quad (29)$$

From the partial derivative of the equilibrium team output y^{SEQ} with respect to the reciprocity intensity,

$$\frac{\partial y^{SEQ}}{\partial \gamma_i}=\frac{8k_i^2(k_i^2+k_j^2)^4[2(k_i^2+k_j^2)^3-\gamma_jk_j^4]}{\{[2(k_i^2+k_j^2)^2-\gamma_jk_j^2]-2\gamma_ik_i^2(k_i^2+k_j^2)^2\}^2} \quad (30)$$

and

$$\frac{\partial y^{SEQ}}{\partial \gamma_j}=\{[2(k_i^2+k_j^2)^2-\gamma_jk_j^2][2(k_i^2+k_j^2)^3-\gamma_jk_j^4]+2k_i^2(k_i^2+k_j^2)^2[2(k_i^2+k_j^2)^2+k_j^2(\gamma_i-\gamma_j)]\}$$

$$\times\frac{4k_j^2(k_i^2+k_j^2)^2}{\{[2(k_i^2+k_j^2)^2-\gamma_jk_j^2]-2\gamma_ik_i^2(k_i^2+k_j^2)^2\}^2} \quad (31)$$

For $2(k_i^2+k_j^2)^2-k_j^2\gamma_j>0$ and $2(k_i^2+k_j^2)^3-\gamma_jk_j^4>0$, in (30) and (31) we can obtain $\frac{\partial y^{SEQ}}{\partial \gamma_i}>0$ and $\frac{\partial y^{SEQ}}{\partial \gamma_j}>0$.

Therefore, in the sequential game, the team output is also an increasing function with respect to the reciprocity intensity. The greater the reciprocity intensity will lead to a higher team output. The reciprocity surely promotes the Pareto improvement of the team production.

6. Comparisons between Simultaneous Game and Sequential Game

6.1 Comparison of Equilibrium Efforts

On one hand, for the leading mover agent i in the sequential game, from the above (15) and (23), we can draw that

$$x_i^{SEQ}-x_i^{SIM}=\frac{\gamma_jk_ik_j{}^2[2(k_i^2+k_j^2)^2-\gamma_jk_j^2][2(k_i^2+k_j^2)^2+k_j^2(\gamma_i-\gamma_j)]}{\{[2(k_i^2+k_j^2)^2-\gamma_jk_j^2]^2-2\gamma_ik_i^2(k_i^2+k_j^2)^2\}[2(k_i^2+k_j^2)^2-\gamma_ik_i^2-\gamma_jk_j^2]} \quad (32)$$

where because of $2(k_i^2+k_j^2)^2-(k_i^2\gamma_i+k_j^2\gamma_j)>0$, we get $2(k_i^2+k_j^2)^2-\gamma_j k_j^2>0$ and $2(k_i^2+k_j^2)^2+k_j^2(\gamma_i-\gamma_j)>0$, and furthermore from (23) and $x_i>0$, we get $[2(k_i^2+k_j^2)^2-k_j^2\gamma_j]^2-2\gamma_i k_i^2(k_i^2+k_j^2)^2>0$, by which $x_i^{SEQ}-x_i^{SIM}>0$. Then

$$x_i^{SEQ}>x_i^{SIM} \tag{33}$$

Clearly, for the leading mover agent i, the effort selected in the sequential game is higher than that selected in the simultaneous game.

On the other hand, for the following mover agent j, according to (16) and (24),

$$x_j^{SEQ}-x_j^{SIM}=\frac{\gamma_j^2 k_i^2 k_j^{\,3}[2(k_i^2+k_j^2)^2+k_j^2(\gamma_i-\gamma_j)]}{\{[2(k_i^2+k_j^2)^2-\gamma_j k_j^2]^2-2\gamma_i k_i^2(k_i^2+k_j^2)^2\}[2(k_i^2+k_j^2)^2-\gamma_i k_i^2-\gamma_j k_j^2]} \tag{34}$$

where from the above condition, we can find $2(k_i^2+k_j^2)^2+k_j^2(\gamma_i-\gamma_j)>0$, $[2(k_i^2+k_j^2)^2-k_j^2\gamma_j]^2-2\gamma_i k_i^2(k_i^2+k_j^2)^2>0$ and $[2(k_i^2+k_j^2)^2-k_j^2\gamma_j]^2-2\gamma_i k_i^2(k_i^2+k_j^2)^2>0$. Then, we obtain $x_j^{SEQ}-x_j^{SIM}>0$. So,

$$x_j^{SEQ}>x_j^{SIM} \tag{35}$$

Therefore, for the following mover agent j, the effort selected in the sequential game is also higher than that selected in the simultaneous game.

Based on the above two aspects, in the sequential game, both the leading and the following mover will choose higher efforts than that in the simultaneous game. If we can let agents select their efforts sequentially instead of simultaneously, it will promote agents to choose higher efforts. Moreover, the moving sequence is not important, because both the leading and the following mover will choose higher efforts.

6.2 Comparison of Team Output

According to (19) and (29), we have

$$y^{SEQ}-y^{SIM}=\frac{4\gamma_j k_i^2 k_j^2(k_i^2+k_j^2)^2[2(k_i^2+k_j^2)^2+k_j^2(\gamma_i-\gamma_j)]}{[2(k_i^2+k_j^2)^2-\gamma_i k_i^2-\gamma_j k_j^2]\{[2(k_i^2+k_j^2)^2-\gamma_j k_j^2]^2-2\gamma_i k_i^2(k_i^2+k_j^2)^2\}} \tag{36}$$

Be similar to (34), we get $y^{SEQ}-y^{SIM}>0$. Thus,

$$y^{SEQ}>y^{SIM} \tag{37}$$

by which the equilibrium team output in the sequential game is higher than that in the simultaneous game.

6.3 Comparative Discussion

According to both equilibrium efforts and team output in the team production, if we can let agents move sequentially instead of simultaneously, agents will choose higher efforts, and the team will get a higher output. However, from (32), (34) and (36), if the final mover in the sequential game agent j is purely selfish, by which there is $\gamma_j=0$, agents will not choose higher efforts and the team will not get a higher output. Clearly, the important prerequisite that the sequential game can promote the Pareto improvement of the team efficiency is that the final mover must be reciprocal. If the final mover who is reciprocal sees that the leading mover chooses a high effort, he will in return choose a high effort.

Moreover, the leading mover, either reciprocal or purely selfish, knows that if he chooses a high effort then the following reciprocal mover will also choose a high effort in return, and hereby will choose a high effort consciously because the high effort of the following mover will increase the team output and thus improve the utility of the leading mover. Consequently, as long as the final mover is reciprocal, agents will always choose high efforts, and hereby get a higher team output. Therefore, only if the final mover is reciprocal, the sequential game can promote the team efficiency, and the moving sequence except the final does not change the results.

It also explains the implication of the principal in the team production. In many cases, the principal neither joins the production, which is determined by reasons such as the principal's identification and specialization, nor supervises agents, which is determined by reasons such as the high monitoring costs and specialization, but can determine the action timing of agents.

The above analysis shows that as long as the principal assigns the moving sequence of agents randomly, he can motivate the agents to choose higher efforts, obtain a higher team output, and hereby promote the Pareto improvement of team production, because when the final mover is reciprocal, the game timing of other agents is not important, which has be illustrated in the above.

In the team production, the principal is usually seen as motivating or supervising agents as in Alchian and Demsetz (1972) et al. In fact, the motivation plays the role through appropriate arrangements, which is irrelative with whether the principal exists or not, and the system of motivation may be not designed and implemented by the principal, which has been illustrated by the incentive mechanisms to promote team cooperation provided by the McAfee and McMillan (1991) et al. The principal often cannot supervise agents, because they cannot watch agents working all the time, and even if he does watch agents working all the time, he cannot judge whether agent work hard or not because of specialization. The above shows that the significance of the principal in the team production is to arrange the moving sequence of agents to ensure that agents move sequentially instead of simultaneously and the final mover is reciprocal, which can greatly improve the team efficiency. Moreover, compared with the usual incentives or supervision, it is also easier for the principal to operate and implement.

7. Comparisons between Reciprocity and Inequity Aversion

Huck and Biel (2006), based on Fehr-Schmidt model, studied how inequity aversion emphasizing fairness of the behavioral outcome influenced the equilibrium efforts and team output. Based on the Rabin model, the above analysis studies the effect of reciprocity emphasizing fairness of the behavioral intention. We will compare the effects of inequity aversion and reciprocity on the equilibrium efforts and team output.

7.1 Difference in Simultaneous Game

Huck and Biel (2006) found that inequity aversion would lead to the consistent effect in the simultaneous game. The more productive agent would decrease his own effort, while the less productive agent would enhance his own effort. The extent how agents changed efforts was a decreasing function with respect to own inequity aversion intensity, but was an increasing function with respect to the inequity aversion intensity of peers. So, inequity aversion may increase or decrease the team output. If and only if the ratio of productivities was greater than the inverse ratio of the inequity aversion intensity, inequity aversion would improve team output.

The above indicates that the impact of reciprocity is more obvious and more monotonic than that of inequity aversion. Whether agents are high or low productive, reciprocity will always enhance the equilibrium efforts. In addition, reciprocity will not only improve own effort, but also will improve the efforts of peers, even if peers are purely selfish. The extent how agents change efforts is an increasing function with respect to own reciprocity intensity and those of peers. Moreover, no matter what's the relationship between the productivities and the reciprocity intensity, whenever at least one agent is reciprocal, the team output will increase.

7.2 Difference in Sequential Game

Huck and Biel (2006) found that inequity aversion would result in the commitment effect in the sequential game. For agents with inequity aversion, the sequential game could promote the Pareto improvement of the team efficiency under strict conditions. However, the moving sequence of agents had an important impact on the team efficiency. Only when the less productive agent move firstly and the productivities of every agents were not too different, the sequential game would be able to promote the Pareto improvement of the team efficiency.

The above indicates that reciprocity can have more substantial improvements in team efficiency under looser conditions. Compared with the simultaneous game, in the sequential game each agent will enhance his own effort, and thus the team will get a higher output. So, the sequential game can further promote the Pareto improvement of the team efficiency. There is only one prerequisite, which requires that the final mover is reciprocal. The game timing of other agents is not important as long as the final mover is reciprocal. It is also irrelative with the productivities of agents. This is very interesting. Huck and Biel (2006) studied the inequity aversion, but found that the condition to ensure the sequential game could promote the Pareto improvement of team production required that agents moved in descending sequence of the productivities instead of the inequity aversion intensity.

The above also indicates that the condition to ensure that the sequential game can promote the Pareto improvement of team production is that the final mover is reciprocal, which is irrelative with the productivities of agents. Clearly, comparing with that of inequity aversion, the influence of reciprocity is more consistent.

8. Conclusions

The above explores the inherent mechanism how reciprocity influences the team efficiency under different game timing, makes comparative analysis with that of selfish preference and inequity aversion respectively, and draws the following conclusions. Firstly, reciprocity can promote the Pareto improvement of the team productivity. Reciprocity can improve own effort, and will also improve the effort of peers, even if peers are not reciprocal but selfish, and hereby will enhance the team output. Secondly, compared with that under simultaneous game, the degree that reciprocity promotes the Pareto improvement of the team productivity under the sequential game is greater. As long as the final mover is reciprocal, the sequential game can promote the Pareto improvement of the team efficiency to a greater extent, even if agents except the final take actions randomly. The moving sequence of agents except the final does not change the results. Thirdly, the conditions under which reciprocity can promote the Pareto improvement of the team efficiency are looser than that of inequity aversion. Whenever at least one agent is reciprocal in the simultaneous game and the final mover is reciprocal in the sequential game, reciprocity can promote the Pareto improvement of the team efficiency.

Although inequity aversion can also promote the Pareto improvement of the team efficiency, there are very strict restrictions including the ratio of productivities is greater than the inverse ratio of inequity aversion intensity in the simultaneous game, and the less productive agent move firstly and the productivities of every agent are not too different in the sequential game. Therefore, in addition to the usual incentive and supervision, there are also two important implication of the principal in working team. First, the principal should screen and select employees with reciprocity to form the team because the reciprocators will raise the efforts of their own and those of peers, and hereby provide a higher team output. Second, the principal should arrange the moving sequence of employees to ensure that employees do not act simultaneously and the final mover is reciprocal, which can further promote the Pareto improvement of the team production.

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