Street as a Stage: A Model of Dynamic Provision of Public Goods under the Threat of Protest

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Abstract

It has become increasingly common that the politically and economically weak citizens use protest as a channel through which they express their dissatisfaction with the policies engendered by the elites. This paper aims to provide a better understanding on this issue by using differential games to study whether protest could trigger some changes in public good investment under uncertainty. In this paper, we show that protest on its own is not sufficient for inducing the elites to invest more in public goods. Instead, the elites -- the group with political power -- choose policy to increase their income and to directly transfer resources from the rest of the society to themselves.

Keyword: protest, public goods, uncertainty, differential games, class conflict

1. Introduction

When the group with political power has preference over inefficient policies, this translates into inefficient economic institutions (Acemoglu, 2006). Even though the dictatorship regime is inefficient or inappropriate, the elites still have an incentive to preserve the system as it is, while other group in the society will fight to induce changes in the policies. In this paper, given that the policies chosen by the elites -- the group with political power -- are inefficient and have impact on economic performance, we study the roles played by collective actions such as protest and revolution in influencing the policies chosen by the elites.

To many people, the word `protest' has connotations of bothersome activity as its occurrence, in many instances, could threaten the economic and political order. Lipsky (1968) defined protest as “a mode of political action oriented toward objection to one or more policies or conditions, characterised by showmanship or display of unconventional nature, and undertaken to obtain rewards from political or economic systems”. It is perceived as a mean through which people, who lack *de jure* power and are unsatisfied with policies, express their dissatisfaction (Muller and Opp, 1986). Lohmann (1993) argues that, on several occasions, major shifts in policies are preceded by different forms of political actions, such as petitions, demonstrations and riots, which allow people to express their dissatisfaction with the status quo.

Despite the risk of imprisonment or exile, and a small chance of achieving ground breaking changes, protests and other forms of political actions have been the mean widely used by people throughout the history. Reiss (2007) documents a series of case studies which discuss the emergence of protests and organised public demonstrations between the nineteenth century and the end of the twentieth century.

In the twenty-first century, protests are still commonly used and have indeed become a global phenomenon, although factors that lead people to protest could be diverse. In this paper, we view protest as being a consequence of government's public good policies.

At present, there is a high tendency in many countries that the governments choose policies which generate *ad hominem* benefits to the elite instead of choosing policies like public good provision which yield diffuse benefits to the entire population since, from the point of view of the elites, it is wasteful to devote a large amount of economy's resources to investment in public goods, whose benefits are mostly enjoyed by the citizens (Lizzeri and Persico, 2004). As a result, public good provision, in these countries could be limited.

When the politically and economically weak groups are adversely affected by the policies determined by the elites, protest could help pave way for some changes and open up opportunities for them. This paper is devoted to provide a better understanding of the concerns facing the politically and economically weak groups of people.

In this paper, we consider an economy consisted of two groups of agents: elites and non-elites. We suppose that elites are the enfranchised group, which makes resource allocation decision for the economy, while the non-elites are the disenfranchised group. We use differential games to study the dynamic interaction between these two groups over time. The non-elites value the consumption of public goods. Since the investment in public good generates diffuse benefits, while the transfer to elites creates *ad hominem* benefits, the elites have a strong incentive to directly transfer the resources from the rest of the society to themselves so they under invest in the public goods. In this model, by being deprived from their public good consumption, the non-elites could choose to engage in costly protest in order to put pressure on the elites to increase the investment in public goods. Since the non-elites supply labour into the production of national output, taking part in the protest diverts inputs away from production. To make this threat of protest credible, it is very important that the non-elites precommit themselves by organising and carrying out the actions before the elites make their decisions.

When the threat of protest becomes credible, the elites then face a trade-off: on one hand, investing more in public goods implies that less resources could be directly transferred to themselves; on the other hand, by not increasing investment in public goods, lower level of production could result. Facing with uncertainty of investment in public good and their preference for direct transfer of resources to themselves, non-elites' engagement in protest alone is not sufficient to trigger policy shift by the elites. Our results show that only when the revolution is looming because the level of public good is too low that the elites start to increase their investment in public goods.

The remainder of the paper is structured as follows. Section 2 is devoted for reviewing the related literature both in political science and in economics. Section 3 describes the model environment. In Section 4, we present the method applied to solve the model. Section 5 presents the sensitivity analysis, while Section 6 concludes.

2. Related Literature

In the economic literature, there are very few papers which looked at the micro-foundation of protest. Buenrostro et al. (2007) use a game-theoretic approach to analyse the state's response to protest movements and its impact on potential protesters. They use a reputation model by Kreps and Wilson (1982) to explain different state's responses to protest, where they allow the state to be either tough or weak.

With more than one protest group, a weak state may choose not to give in to protesters in order to build a reputation for being tough and thus deter other groups from protesting.

Similar to Buenrostro et al. (2007), in this paper, we study the government's response to protest and how the non-elites could use protest to bias policy on public goods in their favour. Unlike in Buenrostro et al. (2007) which allow for heterogeneity across different protest groups, in this paper, we assume that the citizens are homogenous. Moreover, while in Buenrostro et al. (2007), there is incomplete information about each player's type, in our model, there is complete information so we abstract from the signalling of information between protesters and the government.

We now proceed to discuss the government's or political leader's responses to political movement. The question that arises is whether it is rational for the political leaders to ever respond to political actions. While DeNardo (1985) referred to the power in numbers, the response of political leaders to political actions seems to vary over time and across societies in a haphazard fashion. Tarrow (1991) concludes that social scientists are aiming at a moving target when analysing collective political actions.

How does the model proposed in this paper fit in the existing literature? In this paper, we consider a situation in which the non-elites have a limited access to public goods.

In order for them to make sure that their voice is heard, they could choose to take part in the protest. As argued by Olson (1965) and other related papers that, since collective action is usually plagued by a free-rider problem, the threat of collective action might not become credible in bringing about policy changes.

Before the threat of protest becomes a reality and hence, before the government ruled by the elites has to deal with this threat, the non-elites need to overcome the potential collective action problem inherent in coordinating participation in the protest activity. In our paper, we do not model how the collective action problem is solved[1], specifically there is neither coercion nor some other special device to make individual non-elites act in their common interest. We assume along the lines of Acemoglu and Robinson (2006) that both elites and non-elites have solved their collective action problems because this helps us making our modelling exercises tractable and this gives us justification in treating both groups collectively and referring to “the elite” and “the citizen” and examining an equilibrium stemming from interactions between these two groups.

The other closely related strand of literature is the one on enfranchisement. Acemoglu and Robinson (2000, 2001) and Conley and Temimi (2001) introduce civil unrest or threat of revolution and study the disenfranchised group could gain the right to vote by effectively threatening the social order and hence the position of the enfranchised group. A crucial issue is whether the possibility of revolution or civil disobedience is so serious to, by itself, convince the elite to extend the franchise. In our paper, given that it is rational for the disenfranchised non-elites to engage in costly threats of revolution in order to gain access to more share of aggregate resources they have contributed through a higher public good consumption, we study the dynamic interaction between the elite and the citizen over time.

3. The Model

In this section, we present the model which adopts a differential game approach[2]. We consider an economy consisting of elites and non-elites, where the elites are the group with political power. We suppose that the production function of national output, requires two types of inputs, investment in public good and labour. Production function is given by

$$P(u,v) = Auv^2, \tag{1}$$

where u denotes the investment in the public good by the elites and v denote the amount of time the non-elites spend working. It can be seen that the marginal product of u is larger than that of v if $v<1$. In this case, the major contribution to the production is their investment in public good. The value of public good, x, is assumed to follow a stochastic differential equation:

$$dx(t) = \left(u(t) - \delta x(t)\right)dt + \sigma x(t)dW(t), \tag{2}$$

where $\delta > 0$ is the depreciation rate and $W(t)$ is a standard Wiener process[3]. Suppose that the elites' utility from public good consumption is given by

$$U_e(x) = ax - bx^2,$$

where a and b are positive. It can be seen that $U_e(x)$ is negative if x is sufficiently large, i.e., $x > \frac{a}{b}$. Our discussion is therefore restricted to some specific public goods. The objective functional for the elites is defined by

[1] A rich empirical literature has investigated how the collective action problem is solved in practice (Lichbach, 1995; Moore, 1995).

[2] Some applications of differential games in economics can be refered inDockner et al. (2000) and Jorgensen and Zaccour (2006).

[3] The deterministic version of (2) was used in Fershtman and Nitzan (1991), and Wang and Ewald (2010) extended the process used in Fershtman and Nitzan (1991) by adding two types of volatility terms.

$$V_e(x) = \max_u E\left\{\int_t^\infty e^{-r(s-t)}\left(U_e(x(s)) + P(u(s),v(s)) - \frac{u^2(s)}{2}\right)ds \,\middle|\, x(t) = x\right\}, \tag{3}$$

where $\frac{u^2}{2}$ is the individual cost of investment in the public good. Note that the term, $U_e(x)+P(u,v)$, denotes the amount of resources the elites directly transfer to themselves.

Given their time endowment and the observed value of public good, the non-elites can choose the amount of time they spend working in the production of output, $P(u,v)$. The opportunity cost of working is given by $\frac{v^2}{2}$, which suggests that the opportunity cost of working borne by the non-elites is increasing in the time the non-elites devote for production and increases at an increasing rate. The interpretation of the opportunity cost of working is as follows. If the non-elites do not express their dissatisfaction, the elites may not be concerned about their livelihood. The non-elites' utility is given by

$$U_p(x,v) = c\sqrt{x}v,$$

where c is positive. The parameter c is negatively related to the minimum consumption of the public good required by the non-elites, i.e., a smaller c implies a higher minimum consumption. It can be seen that $U_p(x,v)$ is strictly increasing in x and v as well as strictly concave in x. The interpretation for the former is that the utility of non-elites is positively related to v, i.e. the longer the non-elites work, the higher utility they obtain. Moreover, their utility is increasing in the value of public good, i.e. if the non-elites observe that the value of public good becomes smaller, their utility declines. The objective functional for the non-elites is therefore given by

$$V_p(x) = \max_v E\left\{\int_t^\infty e^{-r(s-t)}\left(U_p(x(s),v(s)) - \frac{v^2(s)}{2}\right)ds \,\middle|\, x(t) = x\right\}, \tag{4}$$

4. Solving the Model

To solve the model, the dynamic programming principle leads to the following Hamilton-Jacobi-Bellman (HJB) equations:

$$rV_e(x) = \max_u \left\{ax - bx^2 + Au\left(v^*\right)^2 - \frac{u^2}{2} + \left(u - \delta x\right)V_e'(x) + \frac{\sigma^2 x^2}{2}V_e''(x)\right\}, \tag{5}$$

and

$$rV_p(x) = \max_v \left\{c\sqrt{x}v - \frac{v^2}{2} + \left(u^* - \delta x\right)V_p'(x) + \frac{\sigma^2 x^2}{2}V_p''(x)\right\}, \tag{6}$$

where u^* denotes the optimal investment of the elites in the public good and v^* denotes the optimal amount of time the non-elites spend working. In this paper, we derive the so called stationary feedback Nash-equilibrium strategies, i.e., u^* and v^* are functions of the state x. A necessary condition for v^* is given by

$$v^*(x) = c\sqrt{x}. \tag{7}$$

It can be seen that v^* is independent of u^*, which means that the non-elites are not concerned about the elites' investment in the public good. Instead, their concern is on the value of the public good. It also can be seen that v^* is increasing in x. The interpretation is that the non-elites have fewer incentives to protest if the value of the public good is higher. Substituting v^* into Equation (5) gives

$$rV_e(x) = \max_u \left\{ax - bx^2 + c^2Aux - \frac{u^2}{2} + \left(u - \delta x\right)V_e'(x) + \frac{\sigma^2 x^2}{2}V_e''(x)\right\}. \tag{8}$$

A necessary condition for u^* is given by

$u^*(x) = c^2 Ax + V_e'(x)$.

We substitute the form of u^* into Equation (8) and rearrange all terms. We then obtain

$$\frac{\sigma^2 x^2}{2} V_e''(x) + \frac{1}{2}\left(c^2 Ax + V_e'(x)\right)^2 - \delta x V_e'(x) - rV_e(x) + ax - bx^2 = 0. \tag{9}$$

We make a sophisticated guess and assume that the solution of Equation (9) takes the form of

$V_e(x) = B_2 x^2 + B_1 x + B_0$.

Substituting the above solution form into Equation (9) gives

$$\left[\sigma^2 B_2 + \frac{1}{2}\left(c^2 A + 2B_2\right)^2 - 2\delta B_2 - rB_2 - b\right]x^2 + \left[\left(c^2 A + 2B_2\right)B_1 - \left(r+\delta\right)B_1 + a\right]x + \left(\frac{B_1^2}{2} - rB_0\right) = 0 \tag{10}$$

which therefore implies that

$$\sigma^2 B_2 + \frac{1}{2}\left(c^2 A + 2B_2\right)^2 - 2\delta B_2 - rB_2 - b = 0,$$

$$\left(c^2 A + 2B_2\right)B_1 - \left(r+\delta\right)B_1 + a = 0,$$

$$\frac{B_1^2}{2} - rB_0 = 0.$$

It can be seen that the equation for B_2 is a parabolic equation and there should be two roots. Solving the three equations yields

$$B_2^{\pm} = \frac{-\left(\alpha + 2c^2 A\right) \pm \sqrt{\left(\alpha + 2c^2 A\right)^2 - \left(4c^4 A^2 - 8b\right)}}{4},$$

$$B_1 = \frac{a}{r + \delta + \left(c^2 A + 2B_2\right)},$$

$$B_0 = \frac{B_1^2}{2r},$$

where $\alpha = \sigma^2 - 2\delta - r$. As long as B_2 is determined, B_1 and B_0 are both determined. Note that $B_2^{\pm}$ are well defined if

$$\left(\alpha + 2c^2 A\right)^2 - \left(4c^4 A^2 - 8b\right) \geq 0.$$

In this paper, we consider a public good whose uncertainty is low, more precisely, $\alpha < 0$. Therefore,

$$\left(\alpha + 2c^2 A\right)^2 - \left(4c^4 A^2 - 8b\right) = \alpha^2 + 4\alpha c^2 A + 8b \geq 0,$$

if $\alpha \geq -1$ and $b \geq \frac{c^4 A^2}{2}$. In practice, the depreciation rate and discount rate are much lower than 1, which therefore implies that $\alpha \geq -1$. On the other hand, the elites stop benefiting from the public good when $x > \frac{a}{b}$. A larger b implies that the elites are more unlikely to raise the value of the public good. This case is more interesting than the one for a smaller b.

One may expect that the value function $V_e(x)$ may be concave. Then the guess could be used to determine the value of B_2. Nevertheless, it is not always true. To determine the value of B_2, we apply the finite horizon approximation introduced in Ewald and Wang (2011). We have the following proposition:

Proposition 4.1. If

1. *either* $0 < B_2^- < B_2^+$ *or* $B_2^- < 0 < B_2^+$, *and*
2. B_1 *is positive,*

then $B_2 = B_2^-$ *and* $V_e(x) = B_2 x^2 + B_1 x + B_0$ *is the solution of the stochastic differential game.*

<Proof> See Appendix.

It can be seen in Proposition 4.1 that the value function $V_e(x)$ could be convex, which causes our guess that the value function $V_e(x)$ is concave to be wrong. Therefore, some parameters can lead to a convex value function $V_e(x)$.

5. Sensitivity Analysis

In this section, we analyse the optimal investment of the elites in the public good and study how it is affected by the parameters in the model. Note that

$$u^*(x) = \left(c^2 A + 2B_2\right) x + B_1 .$$

If $B_2 = B_2^- > 0$, then

$$\begin{aligned} c^2 A + 2B_2 &= c^2 A + \frac{-\left(\alpha + 2c^2 A\right) - \sqrt{\left(\alpha + 2c^2 A\right)^2 - \left(4c^4 A^2 - 8b\right)}}{2} \\ &= \frac{-\alpha - \sqrt{\left(\alpha + 2c^2 A\right)^2 - \left(4c^4 A^2 - 8b\right)}}{2} \\ &> 0. \end{aligned}$$

Since

$$\alpha^2 - \left[\left(\alpha + 2c^2 A\right)^2 - \left(4c^4 A^2 - 8b\right)\right] = -4\alpha c^2 A - 8 ,$$

it can be seen that b must not be greater than $\frac{c^4 A^2}{2}$, i.e., b is sufficiently small. As we have indicated earlier, this case is less interesting in the real world since, in the perspective of the elites, the public good is very much profitable. This leads the optimal investment of the elites in the public good to increase in x. Nevertheless, provided that this phenomenon is not caused by the protests by the non-elites, we therefore choose to omit it here.

We now assume that $b > \frac{c^4 A^2}{2}$, which therefore leads $B_2 = B_2^- < 0$. Furthermore, $c^2 A + 2B_2 < 0$, which implies that the optimal investment of the elites in the public good is decreasing in x. The interpretation is as follows. The amount of time the non-elites spend protesting is negatively related to the value of the public good. Moreover, if x is sufficiently large, the production is increasing significantly in x. Therefore, when the value of the public good is sufficiently high, the elites reduce the investment in the public good to obtain higher direct transfer of the resources. We now study how some parameters affect the optimal investment of the elites in the public good. We begin with the parameter c, which is negatively related to the non-elites' minimum consumption of the public good. We have the following proposition:

Proposition 5.1. $u^*(x)$ *is decreasing more significantly in* x *if* c *is smaller.*

<Proof> See Appendix.

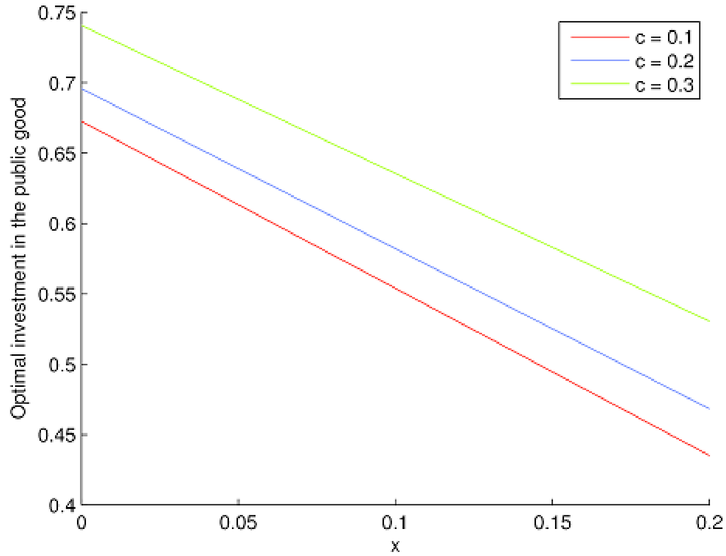

Figure 1: The Optimal Investment of the Elites in the Public Good Under $r = 0.1$**,** $\delta = \sigma = 0.2$**,** $A = 10$ **and** $a = b = 1$

Proposition 5.1 indicates that if the non-elites require higher minimum public good consumption, i.e., the non-elites spend more time protesting, then the elites reduce the investment in the public good significantly when a higher value of the public good is measured. This result is surprising. A possible interpretation is as follows. The elites have fewer incentives to raise the value of the public good since they prefer the direct transfer of the resources to the consumption of the public good. If the non-elites require higher minimum consumption of the public good, i.e., they spend more time protesting, the elites have to invest more in the public good to raise the value of it to placate the angry non-elites. Then the elites have to sacrifice higher direct transfer of the resources, which is not in their will. Therefore, the elites reduce the investment in the public dramatically to receive more transfer of the resources if a higher value of the public good is measured. It can be seen in Figure 1 that a smaller c implies a smaller optimal investment of the elites in the public good. Since $c^2 A + 2B_2 < 0$ is increasing in c and B_1 is increasing in $c^2 A + 2B_2$, B_1 is increasing in c, i.e., a higher minimum consumption of the public good of the non-elites implies that the elites not only reduce the investment significantly in x, but also invest less in the public good.

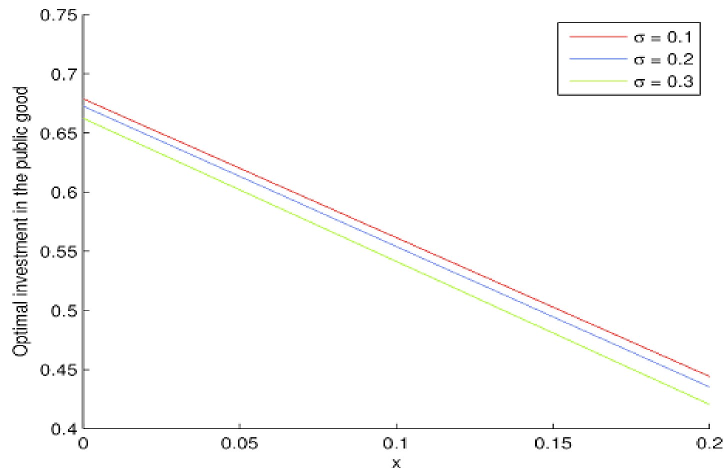

Figure 2: Figure 3: The Optimal Investment of the Elites in the Public Good Under $r = 0.1$, $\delta = \sigma = 0.2$, $A = 10$ **and** $a = b = 1$

We now move on to studying how the uncertainty parameter affects the optimal investment of the elites in the public good. We have the following proposition:

Proposition 5.2. $c^2 A + 2B_2$ *is decreasing in* σ.

<Proof> See Appendix.

The interpretation for Proposition 5.2 is that a higher uncertainty leads the elites to have fewer incentives to invest in the public good. The elites could invest more in the public good in the situation of higher uncertainty, but the uncertainty may soon push the value of the public good at a lower level. Therefore, when the elites observe a higher value of the public good, i.e., the amount of time the non-elites spend protesting is lower, the elites reduce their investment in the public good significantly to receive more direct transfer of the resources. In addition, it can be seen that B_1 is decreasing in σ, which is presented in Figure 2. Therefore, a higher level of uncertainty gives the elites more incentives to reduce the investment in the public good more significantly and fewer incentives to invest more in the public good.

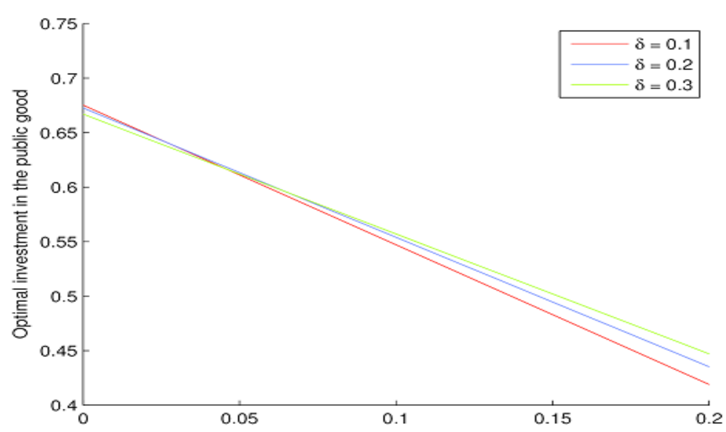

Figure 4: The Optimal Investment of the Elites in the Public Good Under $r = 0.1$, $\delta = \sigma = 0.2$, $A = 10$ **And** $a = b = 1$

The depreciation rate δ is the next concern. We first present the following proposition:

Proposition 5.3. $c^2 A + 2B_2$ *is increasing in* δ.

<Proof> See Appendix.

One may expect that a higher depreciation rate may lead the elites to have less incentives to invest in the public good, but we found the contrary. A possible interpretation is that it takes time to raise the value of the public good at a higher level. Therefore, even though the elites observe a higher value of the public good, they reduce the investment in the public good only slightly. Moreover, in contrast to the effect of the uncertainty on the public good, the effect of the depreciation rate on the public good is slow. Even though the value of the public good declines due to a higher value of the two parameters, the elites do not act the same in the two cases. Note that whether B_1 is increasing or decreasing in δ depends on the parameters chosen. For instance, a sufficiently large A leads B_1 to increase in decrease in δ. In Figure 3, our parameters lead B_1 to decrease in δ. Nevertheless, Proposition 5.3 implies that the elites' optimal investment in the public good is higher given a sufficiently large x if the depreciation rate is larger. Note that $A=10$ is large in our example. A higher A leads to higher production and the elites tend to invest less in the public good to obtain more direct transfer of the resources if the depreciation rate is higher and x is too small. The elites will fight against a higher depreciation rate only if they have some spare resources. On the other hand, if A is small, the elites do not have sufficient amount of resources, which leads them to always invest more in the public good if a higher depreciation rate is observed.

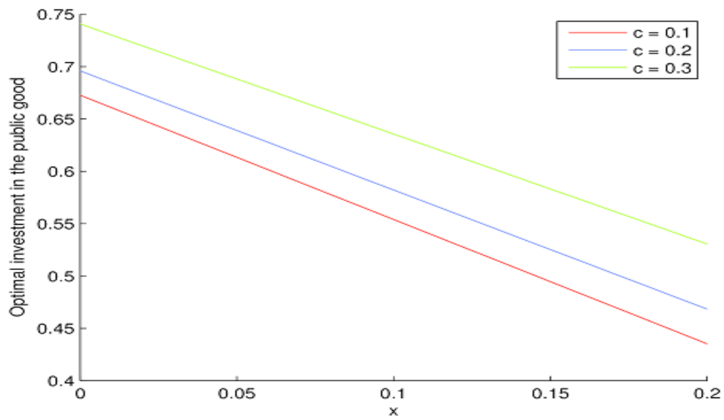

Figure 5: The Optimal Investment of the Elites in the Public Good Under $r=0.1$, $\delta=\sigma=0.2$, $A=10$ **And** $a=b=1$

Last but not least, we present the result of the sensitivity analysis on the parameter b. It can be shown that c^2A+2B_2 is decreasing in b. The interpretation is that the elites can benefit from the public good only if its value lies in a small interval. Therefore, the elites have fewer incentives to raise the value of the public good and it causes the elites to reduce the investment in the public good significantly if a higher value of the public good is observed. Since c^2A+2B_2 is decreasing in b, B_1 is also decreasing in b. Hence, a larger b implies that the elites invest less in the public good. The interpretation is that a larger b implies that the elites' consumption of the public good is smaller, which gives the elites less incentives to raise the value of the public good.

6. Conclusion

In this paper, we study how the politically and the economically weak non-elites could influence the policy choices of the elites, particularly the investment in public good, by taking part in the protest. Our results show that, if a higher value of the public good is observed, a higher uncertainty leads the elites to reduce the investment in the public good significantly. On the other hand, the investment in the public good declines slightly with respect to the value of the public good if the depreciation rate is high. It is interesting to highlight that a larger minimum consumption of public good does not always lead to more investment in the public good even though the non-elites protest longer, which is contrary to the priori belief.

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Appendix

- *<Proof of Proposition 4.1>:*

To prove the proposition, we apply the finite horizon approximation. We consider the following objective functional:

$$\tilde{V}_e(t,x;T) = \max_u E\left\{\int_t^T e^{-r(s-t)}\left(U_e(x(s)) + P(u(s),v(s)) - \frac{u^2(s)}{2}\right)ds \,\middle|\, x(t) = x\right\}, \quad (11)$$

with the constraint Equation (2). $T > 0$ is given sufficiently large. It can be seen that (11) is the finite time version of (3), which $v^*(x) = c\sqrt{x}$ has been substituted in. The terminal condition is given by

$\tilde{V}_e(T,x;T) = 0$.

We apply the dynamic programming principle and obtain the HJB equation

$$\begin{aligned} & r\tilde{V}_e(t,x;T) - \frac{\partial}{\partial t}\tilde{V}_e(t,x;T) \\ & = \max_u \left\{ ax - bx^2 + c^2 Aux - \frac{u^2}{2} + \left(u - \delta x\right)\frac{\partial}{\partial x}\tilde{V}_e(t,x;T) + \frac{\sigma^2 x^2}{2}\frac{\partial^2}{\partial x^2}\tilde{V}_e(t,x;T) \right\} \end{aligned} . \quad (12)$$

A necessary condition for the optimal control is given by

$$u^*(t,x) = c^2 Ax + \frac{\partial}{\partial x}\tilde{V}_e(t,x;T) .$$

Substituting the optimal control into Equation (12) leads to the solution form

$$\tilde{V}_e(t,x;T) = \tilde{B}_2(t;T)x^2 + \tilde{B}_1(t;T)x + \tilde{B}_0(t;T) .$$

We substitute the solution form into Equation (12). We then obtain the following system of ordinary differential equations(ODEs):

$$\tilde{B}_2'(t;T) = -\sigma^2\tilde{B}_2(t;T) - \frac{1}{2}\left(c^2A + 2\tilde{B}_2(t;T)\right)^2 + \left(r + 2\delta\right)\tilde{B}_2(t;T) + b,\ \tilde{B}_2(T;T) = 0 , \quad (13)$$

$$\tilde{B}_1'(t;T) = -\left(c^2A + 2\tilde{B}_2(t;T)\right)\tilde{B}_1(t;T) + \left(r + \delta\right)\tilde{B}_1(t;T) - a,\ \tilde{B}_1(T;T) = 0 , \quad (14)$$

and

$$\tilde{B}_0'(t;T) = -\frac{1}{2}\tilde{B}_1^2(t;T) + r\tilde{B}_0(t;T),\ \tilde{B}_0(T;T) = 0 . \quad (15)$$

It can be seen that $B_2^{\pm}$, B_1 and B_0 are the fixed points for Equations (13)-(15).

We let $s = T - t$ and denote $C_i(s) = \tilde{B}_i(t;T)$, for $i = 1,2,3$.. We then transform Equations (13)-(15) to

$$C_2'(s) = \sigma^2 C_2(s) + \frac{1}{2}\left(c^2A + 2C_2(s)\right)^2 - \left(r + 2\delta\right)C_2(s) - b,\ C_2(0) = 0 , \quad (16)$$

$$C_1'(s) = \left(c^2A + 2C_2(s)\right)C_1(s) - \left(r + \delta\right)C_1(s) + a,\ C_1(0) = 0 , \quad (17)$$

and

$$C_0'(s) = \frac{1}{2}C_1^2(s) - rC_0(s),\ C_0(0) = 0 . \quad (18)$$

We now show how each $C_i(s)$ evolves in $s \in \left[0,T\right]$. Note that

$$C_2'(0) = \frac{c^4A^2}{2} - b,$$
$$C_1'(0) = a > 0,$$
$$C_0'(0) = 0.$$

It can be seen that $C_1(s)$ is increasing at $s = 0$. In the case of $C_2(s)$, we begin with the case of $\frac{c^4A^2}{2} > b$. This leads $C_2(s)$ to decrease at $s = 0$. In addition, $B_2^- < 0 < B_2^+$. The RHS of Equation (16) is equivalent to

$$2C_2^2(s) + \left(\alpha + 2c^2A\right)C_2(s) + \left(\frac{1}{2}c^4A^2 - b\right).$$

It can be seen that for any $C_2^2(s) \in \left(B_2^-, 0\right)$, the above parabolic function is negative, i.e., $C_2(s)$ is decreasing. Since $C_2(s)$ declines at $s = 0$, we can conclude that $C_2(s) < 0$ for all s.

Therefore,

$\lim_{s\to\infty} C_2(s) = \lim_{T\to\infty} \tilde{B}_2(t;T) = \lim_{t\to-\infty} \tilde{B}_2(t;T) = B_2^- < 0$.

On the other hand, since $C_1(s)$ is increasing at $s=0$, there exists an $\varepsilon > 0$ such that $0 < C_1(\varepsilon) < B_1$. Furthermore, $C_2(\varepsilon) < B_2^-$ and $C_1(\varepsilon) < B_1$ lead to $C_1'(\varepsilon) > 0$. Hence, we can conclude that $C_1(s)$ is always increasing and eventually converges to the fixed point B_1 if it is positive, i.e.,

$\lim_{s\to\infty} C_1(s) = \lim_{T\to\infty} \tilde{B}_1(t;T) = \lim_{t\to-\infty} \tilde{B}_1(t;T) = B_1 > 0$.

Regarding $C_0(s)$, since $C_1(s)$ is convergent, $C_0(s)$ must converge to B_0, i.e.,

$\lim_{s\to\infty} C_0(s) = \lim_{T\to\infty} \tilde{B}_0(t;T) = \lim_{t\to-\infty} \tilde{B}_0(t;T) = B_0 > 0$.

With regard to the case of $\frac{c^4A^2}{2} < b$, the proof is analogous to the proof for the above case and therefore, we omit the details here. The finite horizon approximation then implies that

$$\begin{aligned}\lim_{T\to\infty} \tilde{V}_e(t,x;T) &= \lim_{T\to\infty} \tilde{B}_2(t;T)x^2 + \tilde{B}_1(t;T)x + \tilde{B}_0(t;T)\\ &= B_2x^2 + B_1x + B_0\\ &= V_e(x).\end{aligned}$$

- *<Proof of Proposition 5.1>:*

Since $u^*(x)$ is linear in x, we show that the slope of $u^*(x)$ is smaller if c is smaller. If $c=0$, it can be seen that

$$c^2A + 2B_2 = -\frac{\alpha}{2} - \frac{\alpha^2+8b}{2} < 0.$$

Since

$$\begin{aligned}\frac{d}{dc}\left(c^2A+2B_2\right) &= 2cA + \frac{1}{2}\left(-4cA - \frac{8cA\left(\alpha+2c^2A\right)-16c^3A}{2\sqrt{\left(\alpha+2c^2A\right)^2-\left(4c^4A^2-8b\right)}}\right)\\ &= -\frac{2\alpha cA}{2\sqrt{\left(\alpha+2c^2A\right)^2-\left(4c^4A^2-8b\right)}}\\ &> 0,\end{aligned}$$

the slope of $u^*(x)$ is increasing in c. Moreover, $b > \frac{c^4A^2}{2}$ leads to a negative slope of $u^*(x)$. Therefore, the proposition is correct.

- *<Proof of Proposition 5.2>:*

We differentiate the slope of $u^*(x)$ with respect to σ, which yields

$$\frac{d}{d\sigma}\left(c^2A+2B_2\right)=\frac{d}{d\alpha}\left(c^2A+2B_2\right)\frac{d\alpha}{d\sigma}$$

$$=\frac{-\sigma\left[\sqrt{\left(\alpha+2c^2A\right)^2-\left(4c^4A^2-8b\right)}+\left(\alpha+2c^2A\right)\right]}{4\sqrt{\left(\alpha+2c^2A\right)^2-\left(4c^4A^2-8b\right)}}$$

$$<0.$$

Therefore, the proposition is correct.

- *<Proof of Proposition 5.3>:*

We differentiate the slope of $u^*(x)$ with respect to δ, which yields

$$\frac{d}{d\delta}\left(c^2A+2B_2\right)=\frac{d}{d\alpha}\left(c^2A+2B_2\right)\frac{d\alpha}{d\delta}$$

$$=\frac{\sqrt{\left(\alpha+2c^2A\right)^2-\left(4c^4A^2-8b\right)}+\left(\alpha+2c^2A\right)}{\sqrt{\left(\alpha+2c^2A\right)^2-\left(4c^4A^2-8b\right)}}$$

$$>0.$$

Therefore, the proposition is correct.