The Pedagogy of Teaching Probability in an Introductory Business Statistics Course: A More Successful Approach

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Abstract

This study investigates the pedagogy used in teaching probability in an Introductory Business Statistics course at a four-year private university. Considered one of the more difficult and often elusive topics for students to grasp, this study introduces a uniquely structured and highly successful strategy for teaching this topic. The step-by-step outline for statistical independence versus dependence described here results in a better understanding of the material by students, as verified by a marked improvement in their test grades, not only on this topic, but also topics that follow which are based on probability. Classroom-tested, this innovative approach leads to a more thorough comprehension of the concept of probability and provides more positive results than those normally achieved using traditional methods.

Keywords: Probability, Business Curriculum, Business Statistics

1. Introduction

In most business schools at four-year universities, at least one course in Business Statistics is offered and that normally includes probability. In those instances where two courses are offered, the first course is normally divided into four major areas: (1) descriptive statistics; (2) probability theory; (3) discrete probability distributions including the Binomial, Poisson, and Hypergeometric Probability Distributions; and (4) continuous probability distributions including the Uniform and Normal Probability Distributions along with Sampling Distributions of the Mean and the Proportion. Of these four areas, probability theory often provides the most difficulty for students in the course, frequently resulting in a weak understanding of the topic which in turn, results in lower test scores. When this occurs, topics that follow, including probabilities that are associated with discrete and continuous distributions, become more problematic. In the past, university statistics courses have often been criticized for being overly rigid and abstract and for using teaching methods that are more demanding (Hogg, 1991; Willett & Singer, 1992). Sometimes, however, there is a need for organization and structure in problem-solving that some may consider more rigid and abstract, but this structure is essential to a student's overall comprehension of the material. Some studies have also noted the importance of engendering positive affect and perseverance in students (Bude, Van de Wiel, Imbos, Candel, Broers, & Berger, 2007). If students are not confident in their understanding of probability, other topics that follow will also suffer as a consequence. Since making accurate business decisions requires data analysis, and data analysis requires understanding probability, the importance of business students doing well on this topic is critical.

Students often have seriously deficient or confused intuitive ideas about the random phenomenon being studies in probability. The teacher has the difficult task of eradicating these erroneous notions while fostering the sound intuition that leads to self-confidence in understanding theory and application (Trumbo, B.E., 1994). In dealing with probability theory, the typical content of an *Introductory Business Statistics* course includes problem-solving using the addition rules, independence versus dependence, mutually exclusive events, and Bayes' Theorem. Some research on problem solving has shown that students receiving deliberate instruction on how to solve problems become better problem solvers and are able to "think mathematically"(Schoenfeld, 1985).Over the last decade, some statistics education research has emphasized the need for reform in the teaching of statistics (Tishovskaya and Lancaster, 2012).

However, research on the teaching of statistics remains disconnected, fragmented, and difficult to access (Ziffler, Garfield, Alt, Dupuis, Holleque, and Chang, 2008). Despite the growing emphasis on the need for reform in the teaching of statistics and the increase in papers on statistics education, improvements have been slow in coming (Garfield & Ben-Zvi, 2008). Some studies have shown that teaching a conceptual grasp of probability especially, remains a very difficult task, fraught with ambiguity and illusion (Garfield & Ahlgren, 1988). Other studies have shown that ideas of probability and statistics are still very difficult for students to comprehend (Garfield (1995) (Garfield and Ben-Zvi, 2007). Earlier studies suggest a strategy of integrating new authentic assessment techniques that address students' ability to evaluate and utilize statistical knowledge (Chance, 1997; Garfield and Gal, 1999; Garfield & Chance, 2000). An example they suggest is incorporating non-traditional assessment techniques and innovative models that may include concept maps, i.e., graphical representations of an individual's knowledge framework which demonstrate the concepts and how they are related. In this study, a more streamlined approach using an outline as a basis for teaching Probability in an *Introductory Business Statistics* course is used. This method has been found not only to enhance a student's understanding of the material, a result that is confirmed empirically in improved test scores.

2. Methodology

The participants in this study consisted of two samples of students in an undergraduate *Introductory Business Statistics* course in a business college at a mid-size four-year private liberal arts university. The first sample consisted of eighty-seven students in three sections of this course in the fall of 2012 and the spring of 2013 who were taught Probability using the traditional approach normally found in most Business Statistics textbooks (Lind, Marchal, & Wathen, 2012), (Groebner, Shannon, & Fry 2014), (Sharpe, DeVeaux, & Velleman, 2012). With this method, topics such as the addition laws, mutually exclusive events, statistical independence, and marginal, joint, and conditional probabilities are covered in the narrative of the chapter, which is consistent with most of the textbooks in an *Introductory Business Statistics* course. No specific procedure or outline was provided when covering the topic. Earlier research has shown that the events A/B, B/A, and A are all thought to be the same event, and students try to describe them all with the same techniques. It has been noted that effective teaching methods should ultimately draw attention to whatever features distinguish the events as different (Ancker, 2006). The second sample consisted of eighty-nine students in three other sections of this course in the fall of 2013 and the spring of 2014. With this sample, a more structured, stream-lined approach was used that provides the student with a flow chart of procedures to follow when confronted with a probability problem dealing with mutually exclusive, statistical independence, or marginal, joint, and conditional probabilities. This approach consisted of a series of questions with a structured outline for solving problems of the following nature:

1. Find the Probability of **A *or* B**, i.e., P (A U B).
2. Find the Probability of **A *and* B**, i.e., P (A ∩ B).
3. Find the Probability of **A *given* B**, i.e., P (A / B).

To solve any one of the three questions, the following outline is provided:

FIND:

I. ***P(A or B)***

Addition Laws of Probability	
1) $P(A\,or\,B) = P(A) + P(B)$	(Mutually Exclusive Events)
2) $P(A\,or\,B) = P(A) + P(B) - P(A\,and\,B)$	(Non-Mutually Exclusive Events)

II. 1) ***P(A and B)***
2) ***P(A/B)***

Perform a Test of Independence using:	
1) $P(A\,and\,B) = P(A) \cdot P(B)$	If one side of the equation is *equal* to the other side of the equation, then the events are **independent.**
2) $P(A/B) = P(A)$	If they are *not equal*, then the events are **dependent.**

If the events are independent:

1) **Marginal Probability**
2) **Joint Probability**
$P(A \text{ and } B) = P(A) \cdot P(B)$
3) **Conditional Probability**
$P(A/B) = P(A)$

If the events are dependent:

1) **Marginal Probability**
2) **Joint Probability** (Multiplication Laws of Probability)
   a) $P(A \text{ and } B) = P(B) \cdot P(A/B)$
   b) $P(B \text{ and } A) = P(A) \cdot P(B/A)$
3) **Conditional Probability**
   a) $P(A/B) = \dfrac{P(A \text{ and } B)}{P(B)}$
   b) $P(B/A) = \dfrac{P(B \text{ and } A)}{P(A)}$

This outline proved to be an effective way of visually representing how a student can formulate, understand, and then interpret probability problems. The outline visually organizes a student's thinking and summarizes the topics of study thus providing an effective, and efficient approach to answering probability questions in a very logical structured manner. Key words such as ***or***, ***and***, and ***given*** direct the student through the outline when solving a particular probability problem. For example, when asked whether to find P (A or B) the word ***or*** directs one to the Addition Laws of Probability (see Roman numeral I on the outline). Next the student needs to decide whether the events are mutually exclusive or not in order to determine the correct rule to be used. When asked to find the P (A and B), the Joint Probability, or P (A/B), the Conditional Probability, the student then proceeds to Roman Numeral II in the outline which requires a test of independence. Values are substituted into one of these two equations, namely, the equation where the opposite rule of what you are trying to determine is tested. After substituting the appropriate information into the left and right side of the equation, one of two possibilities will occur. First, either one side of the equation will be *exactly equal* to the other side of the equation resulting in the conclusion that the events are independent, or secondly, one side of the equation *will not be exactly equal* to the other side of the equation and the events will be rendered dependent. Once this decision is made as to whether the events are independent or dependent, then the appropriate probability you are trying to find, Marginal, Joint, or Conditional Probability, can be selected in the next group in order to determine the correct formula to be used to solve the specified problem. What initially appears to students as a difficult problem can now be easily solved using this outline. For business problems involving the Addition Laws of Probability as well as the concepts of Independence versus Dependence using Marginal, Joint, and Conditional Probabilities, this outline provides an organization of the topic which makes probability less elusive and complex for the student to understand. Further indication of the positive responses using this outline came from meeting one-on-one with students during office hours and comments which they made on student evaluations, which tended to be very positive.

Some of these comments included:

- "Probability is so much easier to understand using this outline."
- "The probability outline is so helpful in helping me decide how to solve a probability problem."
- "This outline is extremely helpful in determining how to solve problems."
- "Probability is not really that difficult."
- "Believe it or not, I was helping students in other classes."

When each sample of business students was tested on probability, comparisons were made with their test scores. Both tests contained some of the following problems which are indicative of those used in most *Introductory Business Statistics* textbooks. They include problems such as:

- The probabilities of the events A and B are .20 and .30, respectively. The events are not mutually exclusive. The probability that both A and B occur is .15. What is the probability of either A or B occurring?
- A company has studied the number of lost-time accidents occurring at its Brownsville, Texas plant. Historical records show that 6% of the employees had lost-time accidents last year. Management believes that a special safety program will reduce the accidents to 5% during the current year. In addition, it is estimated that 15% of those employees having had lost-time accidents last year will have a lost-time accident during the current year.

What percentage of the employees will have lost-time accidents in both years? What percentage of the employees will have at least one lost-time accident over the 2 year period?

- The credit department of Lion's Department Store in Anaheim, California reported that 30 percent of their sales are cash, 30 percent are paid for by check at the time of the purchase, and 40 percent are charged. Twenty percent of the cash purchases, 90 percent of the checks, and 60 percent of the charges are for more than $50. Ms. Tina Stevens just purchased a new dress that cost $120. What is the probability that she paid cash?
- The probability that Ms. Smith will get an offer on the first job she applies for is .5, and the probability that she will get an offer on the second job she applies for is .6. She thinks that the probability that she will get an offer on both jobs is .15. What is the probability that Ms. Smith gets an offer on the second job given that she receives an offer for the first job? What is the probability that Ms. Smith gets an offer on at least one of the jobs she applies for? What is the probability that Ms. Smith does not get an offer on either of the two jobs she applies for? Are the job offers independent?
- The following table presents income and age characteristics of some stock shareholders in the United States in 1985, as published in the *World Almanac*:

Income

Age (years)

	<15K	15–25K	25–50K	>50K	Totals
<21	1,446	655	140	19	2,260
21–34	226	5,295	3,733	1,839	11,093
35–44	207	1,550	5,711	3,514	10,982
45–54	1,026	1,975	2,370	2,528	7,899
>54	740	3,702	5,922	4,442	14,806
Totals	3,645	13,177	17,876	12,342	47,040

a) What is the probability that a randomly selected stock owner is 21–34 years old?
b) What is the probability that a randomly selected stock owner is <21 years old and has an income of 15–25K?
c) What is the probability that a randomly selected stock owner is <21 years old or has an income of 25–50K?
d) What is the probability that a randomly selected stock owner has an income >50K if he or she is known to be 35–44 years old?
e) What is the probability of randomly selecting a person who is 35–44 years old and who is 45–54 years old?
f) Are the age categories mutually exclusive or independent?

For the sample using the *traditional method* of teaching probability, the average test score was seventy-two with a standard deviation of twenty. Out of the four major areas covered in this course, i.e., descriptive statistics, probability, discrete probability distributions, and continuous probability distributions, the average test score on probability was the lowest of the four test scores, which is normally the case. Business students taught using this traditional method often commented that they found probability to be an elusive, complex, and often very difficult topic to understand. Overall, their lower test grades seemed to reinforce this perception**.** Subsequently, when these students were later tested on discrete and continuous distributions, which are based on probability, their scores tended to be lower than the second sample's scores on these two tests (for comparisons see Table 1 and Table 2).

Table 1: Traditional Method: Descriptive Statistics for Probability & Discrete vs Continuous Distribution Tests

	Mean	Standard Deviation
Probability Test	72	20
Discrete Distributions Test	78	17
Continuous Distributions Test	81	9

The second sample using the *more structured outline approach* was given a similar test to the traditional group using several of the test questions mentioned earlier. The average test score tended to be significantly higher, a mean of eight-one with a standard deviation of fourteen, versus a mean of seventy-two and a standard deviation of twenty using the traditional approach. In addition, students in this sample tended to score higher on the material that followed including probabilities involving discrete distributions and continuous distributions.

The traditional sample had a mean test grade of seventy-eight with a standard deviation of seventeen, while the sample that used the probability outline had an average score of eighty-four with a standard deviation of twelve. With continuous distributions, the sample using traditional methods score an average of eighty-one with a standard deviation of nine and the second sample had a mean score of eighty-eight with a standard deviation of six (See Table 2).

Table 2: Outline Approach: **Descriptive Statistics for Probability & Discrete vs Continuous Distribution Tests**

	Mean	Standard Deviation
Probability Test	81	14
Discrete Distributions Test	84	12
Continuous Distributions Test	88	6

When a hypothesis test was conducted to see if there was a significant difference in the means of the student test scores using the traditional versus the outlined approach, the null hypothesis of equal means was rejected at the .01 level of significance, *indicating a significant difference in the test scores using the two methods.* Using the outlined approach, where student's scores are higher in probability as well as in the distributions that follow, will inevitably provide a stronger basis of understanding when these students take their second Business Statistics course, which generally covers topics such as statistical estimation, hypothesis testing for one and two samples, and hypothesis testing in linear and multiple regression and correlation as well as analysis of variance.

3. Conclusions

The goal of the current study was to test the pedagogy used in teaching probability to business students in an *Introductory Business Statistics* course using the traditional method versus a more streamlined and structured outline method. Students' comments were more positive using this new approach, but more significantly, this approach resulted in a better understanding of probability, which resulted in higher test scores on this topic as well as higher test scores on the topics that followed, i.e., discrete and continuous probability distributions. It appears that when students are taught probability in a more structured, outlined format, they perceive probability problems to be easier and less complicated than when using the traditional approach. Future research might include larger samples over a longer time period to ascertain the overall effectiveness of this new approach.

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